

7.3 Use Similar Right Triangles

ALEKS = Right triangles and geometric mean ; 56% ready

THEOREM

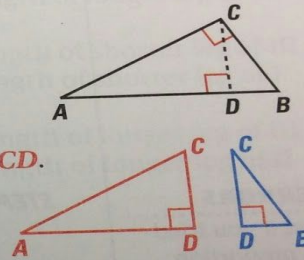
For Your Notebook

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

Proof: below; Ex. 35, p. 456

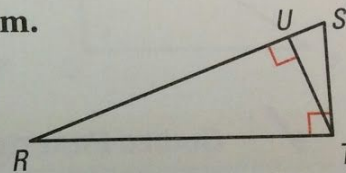


EXAMPLE 1

Identify similar triangles

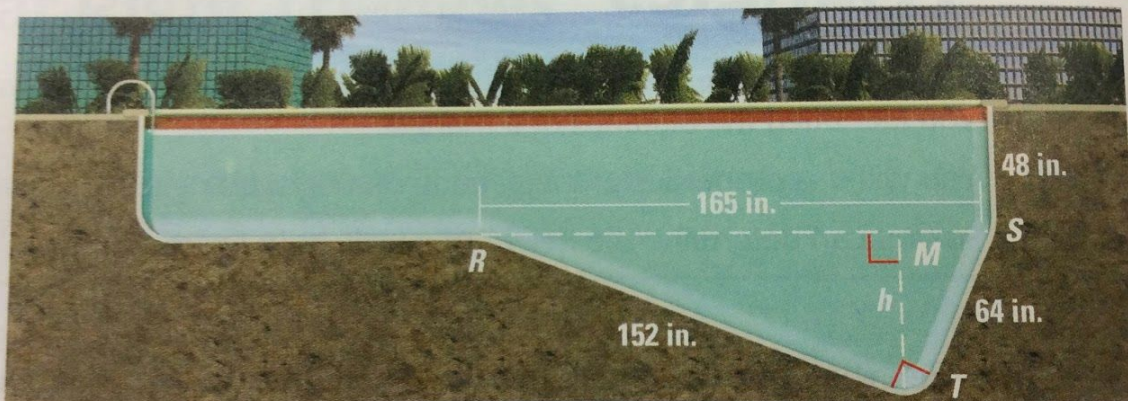
Identify the similar triangles in the diagram.

Solution



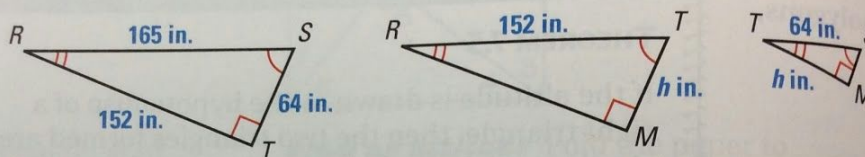
EXAMPLE 2 Find the length of the altitude to the hypotenuse

SWIMMING POOL The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?



Solution

STEP 1 Identify the similar triangles and sketch them.



$$\triangle RST \sim \triangle RTM \sim \triangle TSM$$

STEP 2 Find the value of h . Use the fact that $\triangle RST \sim \triangle RTM$ to write a proportion.

$$\frac{TM}{ST} = \frac{TR}{SR}$$

Corresponding side lengths of similar triangles are in proportion.

$$\frac{h}{64} = \frac{152}{165}$$

Substitute.

$$165h = 64(152)$$

Cross Products Property

$$h \approx 59$$

Solve for h .

STEP 3 Read the diagram above. You can see that the maximum depth of the pool is $h + 48$, which is about $59 + 48 = 107$ inches.

► The maximum depth of the pool is about 107 inches.

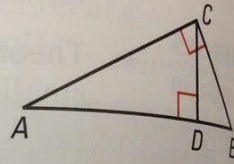
THEOREMS

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

Proof: Ex. 36, p. 456



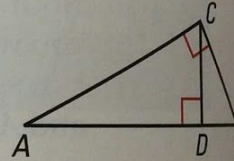
$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

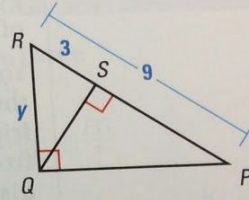
Proof: Ex. 37, p. 456



$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

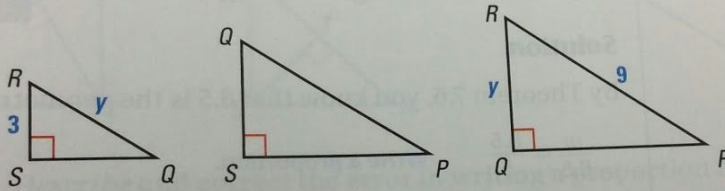
EXAMPLE 3 Use a geometric mean

xy Find the value of y . Write your answer in simplest radical form.



Solution

STEP 1 Draw the three similar triangles.



STEP 2 Write a proportion.

$$\frac{\text{length of hyp. of } \triangle RPQ}{\text{length of hyp. of } \triangle RQS} = \frac{\text{length of shorter leg of } \triangle RPQ}{\text{length of shorter leg of } \triangle RQS}$$

$$\frac{9}{y} = \frac{y}{3} \quad \text{Substitute.}$$

$$27 = y^2 \quad \text{Cross Products Property}$$

$$\sqrt{27} = y \quad \text{Take the positive square root of each side.}$$

$$3\sqrt{3} = y \quad \text{Simplify.}$$

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