4.1 Factors and Prime Factorization

Goal: Write the prime factorization of a number.

Vocabulary

Prime number:

Composite number:

Prime factorization:

Factor tree:

Monomial:

Example 1 *Writing Factors*

A rectangle has an area of 18 square feet. Find all possible whole number dimensions of the rectangle.

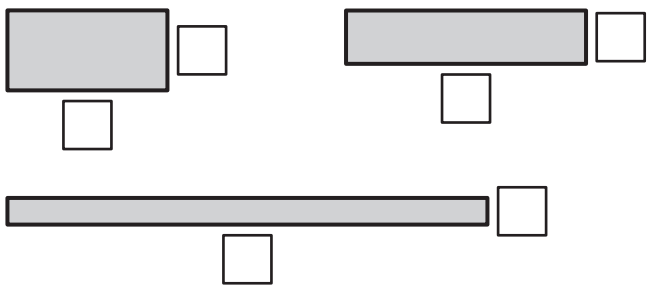
1. Write 18 as a product of two whole numbers in all possible ways.

$\square \cdot \square = 18$ $\square \cdot \square = 18$ $\square \cdot \square = 18$

The factors of 18 are $\square$.

The area of a rectangle can be found using the formula, Area = length × width.

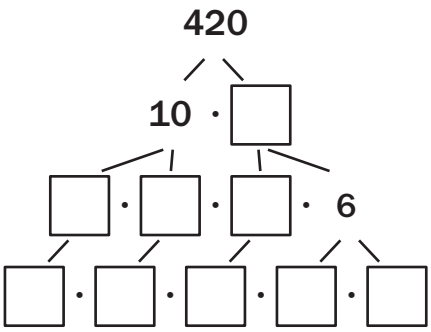
2. Use the factors to find all rectangles with an area of 18 square feet that have whole number dimensions. Then label the given rectangles.



Example 2 *Writing a Prime Factorization*

Write the prime factorization of 420.

One possible factor tree:



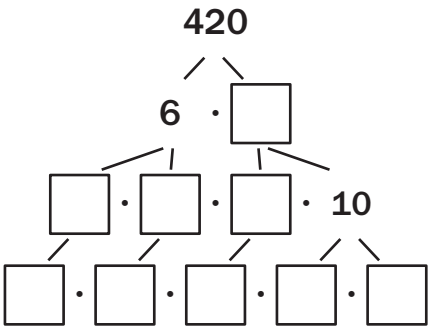
Write original number.

Write 420 as 10 • ☐.

Write 10 as ☐ • ☐. Write ☐ as ☐ • 6 .

Write 6 as ☐ • ☐.

Another possible factor tree:



Write original number.

Write 420 as 6 • ☐.

Write 6 as ☐ • ☐. Write ☐ as ☐ • 10 .

Write 10 as ☐ • ☐.

Both trees give the same result: 420 = ☐.

Answer: The prime factorization of 420 is ☐.

Example 3 *Factoring a Monomial*

Factor the monomial $24x^4y$.

$24x^4y =$ ☐ $\cdot\ x^4y$ Write 24 as ☐.

$=$ ☐ $\cdot\ y$ Write x^4 as ☐.

Checkpoint **Write all factors of the number.**

1. 28	**2.** 48

Tell whether the number is *prime* or *composite*. If it is composite, write its prime factorization.

3. 97	**4.** 117

Factor the monomial.

5. $21n^5$	**6.** $18x^2y^3$

4.2 Greatest Common Factor

Goal: Find the greatest common factor of two or more numbers.

Vocabulary

Common factor:

Greatest common factor (GCF):

Relatively prime:

Example 1 *Finding the Greatest Common Factor*

Volunteers A high school asks for volunteers to help clean up local highways on one Saturday each month. The group of volunteers has 27 freshman, 18 sophomores, 36 juniors, and 45 seniors. What is the greatest number of groups that can be formed if each group is to have the same number of each type of student? How many freshman, sophomores, juniors, and seniors will be in each group?

Solution

Method 1 List the factors of each number. Identify the greatest number that is on every list.

Factors of 27:

Factors of 18:

Factors of 36:

Factors of 45:

The common factors are ____.

The GCF is ____.

Method 2 Write the prime factorization of each number. The GCF is the product of the prime factors.

27 = ____

18 = ____

36 = ____

45 = ____

The common prime factors are ____.

The GCF is ____.

Answer: The greatest number of groups that can be formed is ____. Each group will have 27 ÷ ____ = ____ freshman, 18 ÷ ____ = ____ sophomores, 36 ÷ ____ = ____ juniors, and 45 ÷ ____ = ____ seniors.

✔ ***Checkpoint*** **Find the greatest common factor of the numbers.**

1. 54, 81	**2.** 12, 48, 66

Example 2 ***Identifying Relatively Prime Numbers***

Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.

a. 28, 63 **b.** 42, 55

Solution

a. List the factors of each number. Identify the greatest number that the lists have in common.

Factors of 28: ____

Factors of 63: ____

The GCF is ____. So, the numbers ____ relatively prime.

b. Write the prime factorization of each number.

42 = ____ 55 = ____

The GCF is ____. So, the numbers ____ relatively prime.

✔ ***Checkpoint*** **Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.**

3. 30, 49	**4.** 52, 78

Example 3 ***Finding the GCF of Monomials***

Find the greatest common factor of $16x^2y$ and $26x^2y^3$.

Solution

Factor the monomials. The GCF is the product of the common factors.

$16x^2y =$ ☐

$26x^2y^3 =$ ☐

Answer: The GCF is ☐.

✔ ***Checkpoint*** **Find the greatest common factor of the monomials.**

5. $12x^3$, $18x^2$	**6.** $40xy^3$, $24xy$

4.3 Equivalent Fractions

Goal: Write equivalent fractions.

Vocabulary

Equivalent fractions: _____

Simplest form: _____

Equivalent Fractions

Words To write equivalent fractions, multiply or divide the numerator and the denominator by the same nonzero number.

Algebra For all numbers a, b, and c, where $b \neq 0$ and $c \neq 0$, $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$.

Numbers $\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$ $\qquad$ $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

Example 1 *Writing Equivalent Fractions*

Write two fractions that are equivalent to $\frac{6}{18}$.

Multiply or divide the numerator and the denominator by the _____.

$\frac{6}{18} = \frac{6 \cdot 2}{18 \cdot 2} =$ ☐ Multiply numerator and denominator by 2.

$\frac{6}{18} = \frac{6 \div 3}{18 \div 3} =$ ☐ Divide numerator and denominator by 3.

Answer: The fractions ☐ and ☐ are equivalent to $\frac{6}{18}$.

Checkpoint **Write two fractions that are equivalent to the given fraction.**

1. $\frac{7}{14}$	**2.** $\frac{4}{16}$	**3.** $\frac{10}{25}$

Example 2 ***Writing a Fraction in Simplest Form***

Write $\frac{8}{36}$ in simplest form.

Write the prime factorizations of the numerator and denominator.

$8 =$ ☐ $36 =$ ☐

The GCF of 8 and 36 is ☐.

$\frac{8}{36} = \frac{8 \div \square}{36 \div \square}$ Divide numerator and denominator by GCF.

$= \square$ Simplify.

Checkpoint **Write the fraction in simplest form.**

4. $\frac{3}{18}$	**5.** $\frac{12}{32}$	**6.** $\frac{24}{42}$

Example 3 ***Simplifying a Variable Expression***

Write $\frac{14x^2y}{35x^3}$ in simplest form.

$\frac{14x^2y}{35x^3} = \frac{\square}{\square}$ Factor numerator and denominator.

$= \frac{\square\square\square}{\square}$ Divide out common factors.

$= \square$ Simplify.

Checkpoint **Write the variable expression in simplest form.**

7. $\frac{9a}{15a^2}$	**8.** $\frac{16mn^2}{28n}$	**9.** $\frac{39st^2}{3s^2t}$

4.4 Least Common Multiple

Goal: Find the least common multiple of two numbers.

Vocabulary

Multiple:

Common multiple:

Least common multiple (LCM):

Least common denominator (LCD):

Example 1 *Finding the Least Common Multiple*

Find the least common multiple of 6 and 14.

Solution

You can use one of two methods to find the LCM.

Method 1 List the multiples of each number. Identify the least number that is on both lists.

Multiples of 6:

Multiples of 14:

The LCM of 6 and 14 is ____.

Method 2 Find the common factors of the numbers.

6 = ____

14 = ____

The common factor is ____.

Multiply all of the factors, using each common factor only once.

LCM = ____ = ____

Answer: Both methods get the same result. The LCM is ____.

Example 2 ***Finding the Least Common Multiple of Monomials***

Find the least common multiple $6xy$ and $16x^2$.

$6xy =$ ☐

$16x^2 =$ ☐

LCM = ☐ = ☐

Answer: The least common multiple of $6xy$ and $16x^2$ is ☐.

✔ ***Checkpoint*** **Find the least common multiple of the numbers or the monomials.**

1. 8, 18	**2.** 4, 5, 15
3. $12x$, $18x^2$	**4.** $4xy$, $10xz^2$

Example 3 ***Comparing Fractions Using the LCD***

Summer Sports Last year, a summer resort had 165,000 visitors, including 44,000 water skiers. This year, the resort had 180,000 visitors, including 63,000 water skiers. In which year was the fraction of water skiers greater?

Solution

1. Write the fractions and simplify.

Last year: $\frac{\text{Number of water skiers}}{\text{Total number of visitors}} =$ ☐ = ☐

This year: $\frac{\text{Number of water skiers}}{\text{Total number of visitors}} =$ ☐ = ☐

2. Find the LCD of ☐ and ☐. The LCM of ☐ and ☐ is ☐. So, the LCD of the fractions is ☐.

3. Write equivalent fractions using the LCD.

Last year: $\square = \square = \square$

This year: $\square = \square = \square$

4. Compare the numerators: $\square < \square$, so $\square < \square$.

Answer: The fraction of water skiers was greater $\square$.

Example 4 *Ordering Fractions and Mixed Numbers*

Order the numbers $4\frac{5}{12}$, $\frac{9}{2}$, and $\frac{33}{8}$ from least to greatest.

1. Write the mixed number as an improper fraction.

$$4\frac{5}{12} = \frac{\square}{12} = \frac{\square}{12}$$

2. Find the LCD of $\frac{\square}{12}$, $\frac{9}{2}$, and $\frac{33}{8}$. The LCM of 12, 2, and 8 is $\square$. So, the LCD is $\square$.

3. Write equivalent fractions using the LCD.

$$\frac{\square}{12} = \frac{\square \cdot \square}{12 \cdot \square} = \square \qquad \frac{9}{2} = \frac{9 \cdot \square}{2 \cdot \square} = \square$$

$$\frac{33}{8} = \frac{33 \cdot \square}{8 \cdot \square} = \square$$

4. Compare the numerators: $\square < \square$ and $\square < \square$, so $\square < \square$ and $\square < \square$.

Answer: From least to greatest, the numbers are $\square$, $\square$, and $\square$.