12.1 Angle Relationships

Goal: Classify special pairs of angles.

Vocabulary

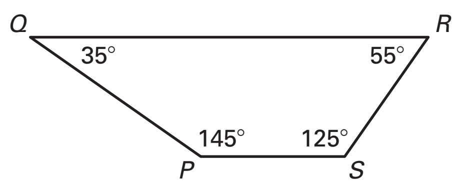
Complementary angles:

Supplementary angles:

Vertical angles:

Example 1 ***Identifying Complementary, Supplementary Angles***

In quadrilateral *PQRS*, identify all pairs of complementary angles and supplementary angles.



Solution

a. Because $m\angle Q + m\angle R =$ ___ + ___ = ___, $\angle Q$ and $\angle R$ are ___ angles.

b. Because $m\angle P + m\angle Q =$ ___ + ___ = ___, $\angle P$ and $\angle Q$ are ___ angles.

c. Because $m\angle R + m\angle S =$ ___ + ___ = ___, $\angle R$ and $\angle S$ are ___ angles.

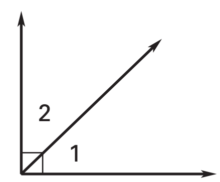
✔ ***Checkpoint*** **Tell whether the angles are *complementary*, *supplementary*, or *neither*.**

1. $m\angle 1 = 37°$ $m\angle 2 = 73°$	**2.** $m\angle 3 = 42°$ $m\angle 4 = 48°$	**3.** $m\angle 5 = 127°$ $m\angle 6 = 53°$

Adjacent angles that form a right angle are complementary. Adjacent angles that form a straight angle are supplementary.

Example 2 ***Finding an Angle Measure***

For the diagram shown, $\angle 1$ and $\angle 2$ are complementary angles, and $m\angle 1 = 46°$. Find $m\angle 2$.



Solution

$m\angle 1 + m\angle 2 = \boxed{}$ Definition of complementary angles

$\boxed{} + m\angle 2 = \boxed{}$ Substitute for $m\angle 1$.

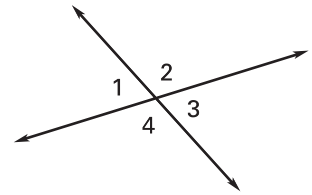
$m\angle 2 = \boxed{}$ Subtract $\boxed{}$ from each side.

✔ ***Checkpoint*** **$\angle 1$ and $\angle 2$ are complementary angles. Given $m\angle 1$, find $m\angle 2$.**

4. $m\angle 1 = 64°$	**5.** $m\angle 1 = 13°$
6. $m\angle 1 = 82°$	**7.** $m\angle 1 = 7°$

Example 3 *Using Supplementary and Vertical Angles*

For the diagram shown, $m\angle 1 = 65°$.
Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



Solution

a. $m\angle 1 + m\angle 2 = \boxed{}$ — $\angle 1$ and $\angle 2$ are supplementary.

$\boxed{} + m\angle 2 = \boxed{}$ — Substitute for $m\angle 1$.

$m\angle 2 = \boxed{}$ — Subtract $\boxed{}$ from each side.

b. $m\angle 3 = \boxed{}$ — Vertical angles have same measure.

$m\angle 3 = \boxed{}$ — Substitute for $m\angle 1$.

c. $m\angle 4 = \boxed{}$ — Vertical angles have same measure.

$m\angle 4 = \boxed{}$ — Substitute for $m\angle 2$.

✔ *Checkpoint*

8. $\angle 1$ and $\angle 2$ are supplementary angles, and $m\angle 1 = 132°$. Find $m\angle 2$.	**9.** $\angle 3$ and $\angle 4$ are supplementary angles, and $m\angle 3 = 23°$. Find $m\angle 4$.
10. In Example 3, suppose that $m\angle 1 = 54°$. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.	

12.2 Angles and Parallel Lines

Goal: Identify angles when a transversal intersects lines.

Vocabulary

Transversal:

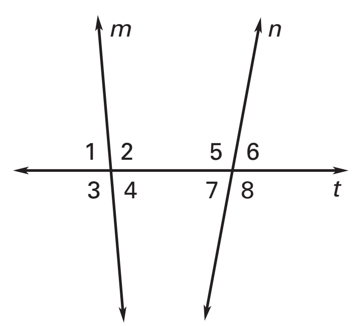
Corresponding angles:

Alternate interior angles:

Alternate exterior angles:

Example 1 ***Identifying Angles***

In the diagram, line t is a transversal. Tell whether the angles are *corresponding*, *alternate interior*, or *alternate exterior* angles.



a. $\angle 1$ and $\angle 5$

b. $\angle 2$ and $\angle 7$

c. $\angle 3$ and $\angle 6$

Solution

a. $\angle 1$ and $\angle 5$ are ______ angles.

b. $\angle 2$ and $\angle 7$ are ______ angles.

c. $\angle 3$ and $\angle 6$ are ______ angles.

✔ ***Checkpoint*** **In Example 1, tell whether the angles are *corresponding*, *alternate interior*, or *alternate exterior* angles.**

1. $\angle 4$ and $\angle 5$	**2.** $\angle 1$ and $\angle 8$	**3.** $\angle 4$ and $\angle 8$

Angles and Parallel Lines

In the diagram, transversal *t* intersects parallel lines *m* and *n*.

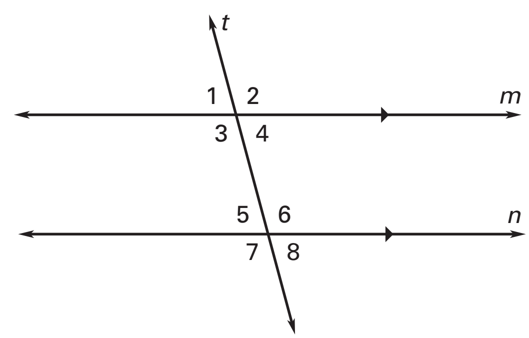
Corresponding angles

$m\angle 1 = \boxed{}$

$m\angle 2 = \boxed{}$

$m\angle 3 = \boxed{}$

$m\angle 4 = \boxed{}$



Alternate interior angles

$m\angle 3 = \boxed{}$

$m\angle 4 = \boxed{}$

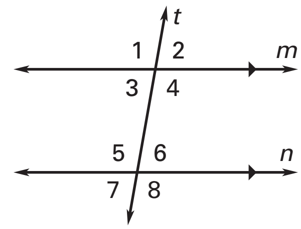
Alternate exterior angles

$m\angle 1 = \boxed{}$

$m\angle 2 = \boxed{}$

Example 2 *Finding Angle Measures*

In the diagram, transversal t intersects parallel lines m and n. If $m\angle 1 = 100°$, find the measures of the other numbered angles.



Solution

$m\angle 5 =$ ______, because $\angle 1$ and $\angle 5$ are ______ angles.

$m\angle 4 =$ ______, because $\angle 4$ and $\angle 5$ are ______ angles.

$m\angle 8 =$ ______, because $\angle 1$ and $\angle 8$ are ______ angles.

$m\angle 2 =$ ______, because $\angle 1$ and $\angle 2$ are ______ angles.

$m\angle 6 =$ ______, because $\angle 2$ and $\angle 6$ are ______ angles.

$m\angle 3 =$ ______, because $\angle 3$ and $\angle 6$ are ______ angles.

$m\angle 7 =$ ______, because $\angle 2$ and $\angle 7$ are ______ angles.

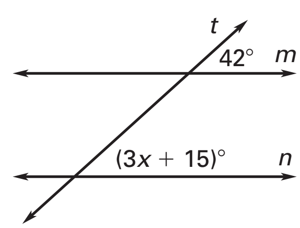
Checkpoint

4. In Example 2, if $m\angle 2 = 85°$, find the measures of the other angles.

If a transversal intersects two lines so that the corresponding angles have the same measure, then the lines are parallel.

Example 3 *Finding the Value of a Variable*

Find the value of x that makes lines m and n parallel.



Solution

The labeled angles in the diagram are corresponding angles. Lines m and n are ______ when the measures are ______.

______ = ______ Set measures equal.

______ = ______ Subtract ______ from each side.

$x =$ ______ Divide each side by ______.

12.3 Angles and Polygons

Goal: Find measures of interior and exterior angles.

Vocabulary

Interior angle: $\boxed{}$

Exterior angle: $\boxed{}$

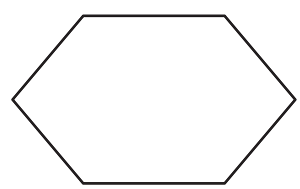
Measures of Interior Angles of a Convex Polygon

The sum of the measures of the interior angles of a convex *n*-gon is given by the formula $(n - 2) \cdot 180°$.

The measure of an interior angle of a regular *n*-gon is given by the formula $\frac{(n - 2) \cdot 180°}{n}$.

Example 1 ***Finding the Sum of a Polygon's Interior Angles***

Find the sum of the measures of the interior angles of the polygon.



Solution

For a convex hexagon, $n = \boxed{}$.

$$(n - 2) \cdot 180° = \left(\boxed{} - 2\right) \cdot 180°$$
$$= \boxed{} \cdot 180°$$
$$= \boxed{}$$

Example 2 ***Finding the Measure of an Interior Angle***

Find the measure of an interior angle of a regular octagon.

Solution

For a regular octagon, $n = 8$.

Measure of an interior angle	=	______	Write formula.
	=	______	Substitute for n.
	=	______	Simplify.

Checkpoint

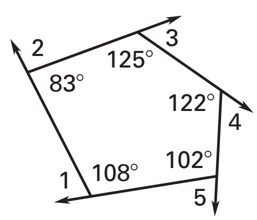
1. Find the sum of the measures of the interior angles of a convex 9-gon.

2. Find the measure of an interior angle of a regular 18-gon.

An interior angle and an exterior angle at the same vertex form a straight angle.

Example 3 ***Finding the Measure of an Exterior Angle***

Find $m\angle 1$ in the diagram.



Solution

The angle that measures ______ forms a straight angle with $\angle 1$, which is the exterior angle at the same vertex.

$m\angle 1 +$ ______ $=$ ______ Angles are supplementary.

$m\angle 1 =$ ______ Subtract ______ from each side.

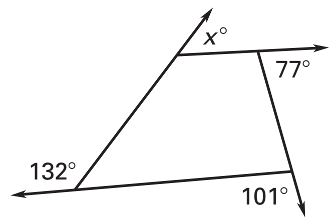
Checkpoint

3. In Example 3, find $m\angle 2$, $m\angle 3$, $m\angle 4$, and $m\angle 5$.

Each vertex of a convex polygon has two exterior angles. If you draw one exterior angle at each vertex, then the sum of the measures of these angles is 360°.

Example 4 *Using the Sum of Measures of Exterior Angles*

Find the unknown angle measure in the diagram.



Solution

$x° + 77° + 101° + 132° = \boxed{}$ Sum of measures of exterior angles of a convex polygon is 360°.

$x + \boxed{} = \boxed{}$ Add.

$x = \boxed{}$ Subtract $\boxed{}$ from each side.

Answer: The angle measure is $\boxed{}$.

Checkpoint

4. Five exterior angles of a convex hexagon have measures 42°, 78°, 60°, 55°, and 62°. Find the measure of the sixth exterior angle.

12.4 Translations

Goal: Translate figures in a coordinate plane.

Vocabulary

Transformation:

Image:

Translation:

Tessellation:

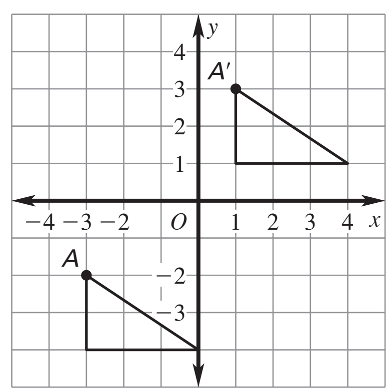
In a translation, a figure and its image are congruent.

Example 1 ***Describing a Translation***

For the diagram shown, describe the translation in words.

Solution

Think of moving horizontally and vertically from a point on the original figure to the corresponding point on the new figure. For instance, you move ____ units to the ____ and ____ units ____ from $A(-3, -2)$ to reach A' ____.

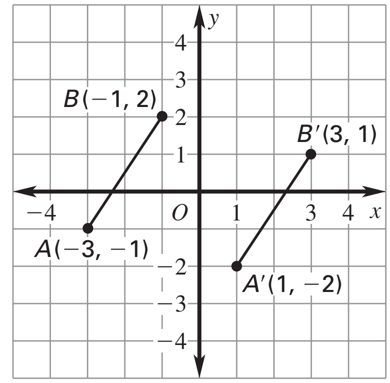


Coordinate Notation

You can describe a translation of each point (x, y) of a figure using the coordinate notation

$$(x, y) \rightarrow (x + a, y + b)$$

where a indicates how many units a point moves horizontally, and b indicates how many units a point moves ____. Move the point (x, y) to the right if a is positive and to the ____ if a is ____. Move the point up if b is positive and ____ if b is ____.



$(x, y) \rightarrow$ ____

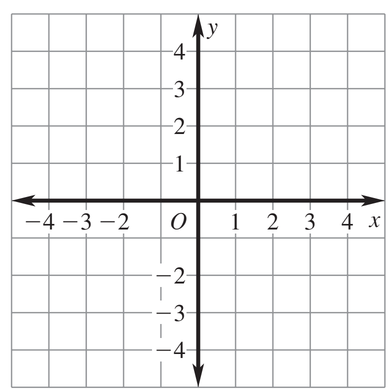
Example 2 *Translating a Figure*

Draw $\triangle ABC$ with vertices $A(-2, 1)$, $B(-1, 4)$, and $C(0, 1)$. Then find the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y - 5)$, and draw the image.

Solution

First draw $\triangle ABC$. Then, to translate $\triangle ABC$, ____ to the x-coordinate and ____ from the y-coordinate of each vertex.

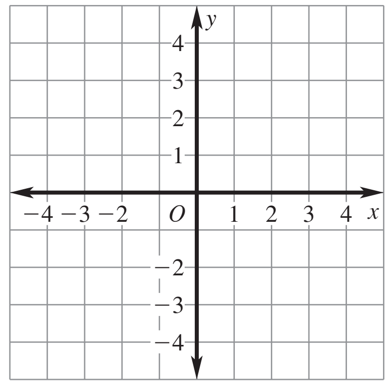
Original		**Image**
(x, y)	$\rightarrow$	$(x + 4, y - 5)$
$A(-2, 1)$	$\rightarrow$	A' ____
$B(-1, 4)$	$\rightarrow$	B' ____
$C(0, 1)$	$\rightarrow$	C' ____



Finally, draw $\triangle A'B'C'$. Notice that each point on $\triangle ABC$ moves ____ units to the ____ and ____ units ____.

✔ Checkpoint

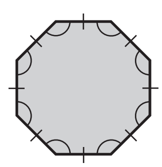
1. Draw quadrilateral *PQRS* with vertices $P(-4, -1)$, $Q(-1, 0)$, $R(-2, -3)$, and $S(-4, -4)$. Then find the coordinates of the image after the translation $(x, y) \rightarrow (x + 6, y + 5)$, and draw the image.



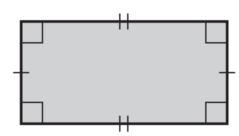
Example 3 *Creating Tessellations*

Tell whether you can create a tessellation using only translations of the given polygon. If you can, create a tessellation. If not, explain why not.

a.

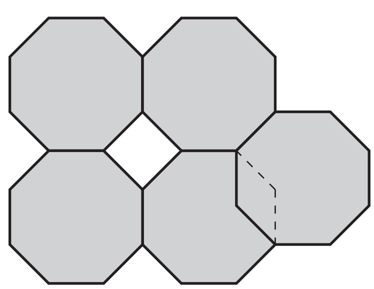


b.

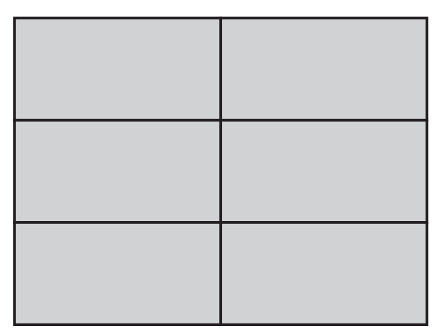


Solution

a. You ______ translate a regular octagon to create a tessellation. Notice in the design that there ______ gaps and overlaps.



b. You ______ translate the rectangle to create a tessellation. Notice in the design that there ______ gaps or overlaps.



12.5 Reflections and Symmetry

Goal: Reflect figures and identify lines of symmetry.

Vocabulary

Reflection:

Line of reflection:

Line symmetry:

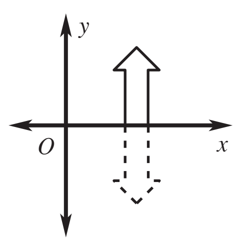
Line of symmetry:

In a reflection, a figure and its image are congruent.

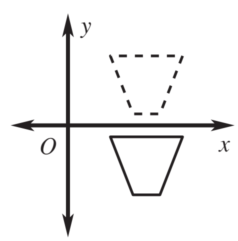
Example 1 *Identifying Reflections*

Tell whether the transformation is a reflection. If so, identify the line of reflection.

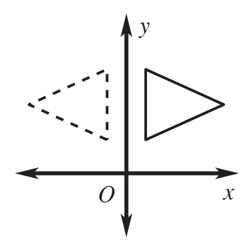
a.



b.



c.



Solution

a.

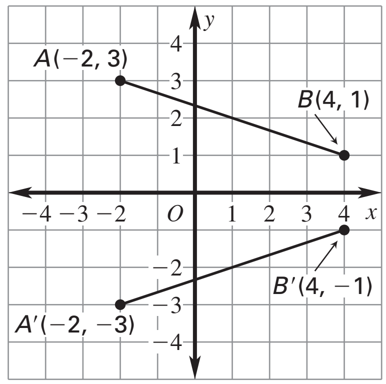
b.

c.

Coordinate Notation

You can use coordinate notation to describe the images of figures after reflections in the axes of a coordinate plane.

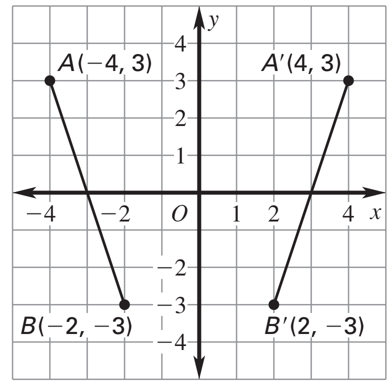
Reflection in the *x*-axis



Multiply the *y*-coordinate by −1.

$(x, y) \rightarrow$ $\boxed{}$

Reflection in the *y*-axis



Multiply the *x*-coordinate by −1.

$(x, y) \rightarrow$ $\boxed{}$

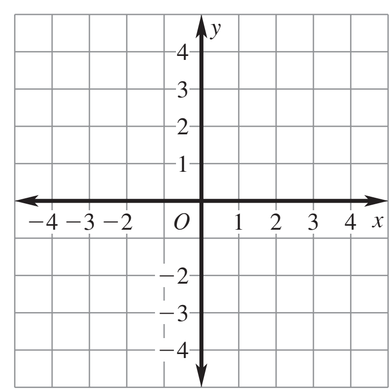
Example 2 ***Reflecting a Triangle***

Draw $\triangle ABC$ with vertices $A(2, 2)$, $B(2, 5)$, and $C(4, 1)$. Then find the coordinates of the vertices of the image after a reflection in the *x*-axis, and draw the image.

Solution

First draw $\triangle ABC$. Then, to reflect $\triangle ABC$ in the *x*-axis, multiply the *y*-coordinate of each vertex by $\boxed{}$.

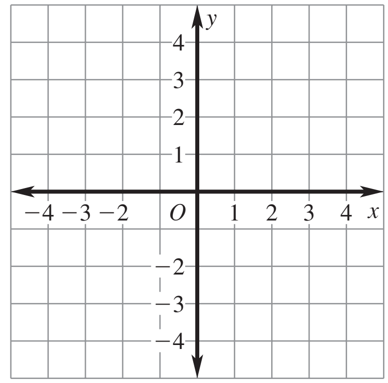
Original		**Image**
(x, y)	$\rightarrow$	$\boxed{}$
$A(2, 2)$	$\rightarrow$	A' $\boxed{}$
$B(2, 5)$	$\rightarrow$	B' $\boxed{}$
$C(4, 1)$	$\rightarrow$	C' $\boxed{}$



Finally, draw $\triangle A'B'C'$.

✔ Checkpoint

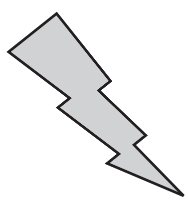
1. Draw $\triangle ABC$ with vertices $A(-4, -3)$, $B(-4, 4)$, and $C(-1, -3)$. Then find the coordinates of the vertices of the image of $\triangle ABC$ after a reflection in the y-axis, and draw the image.



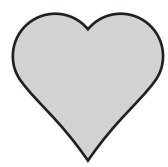
Example 3 *Identifying Lines of Symmetry*

Draw the lines of symmetry on the figure. Tell how many lines of symmetry the figure has.

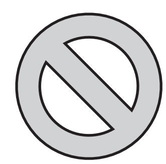
a.



b.



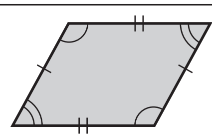
c.



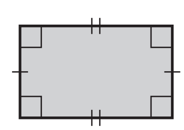
✔ Checkpoint

Draw the lines of symmetry on the figure. Tell how many lines of symmetry the figure has.

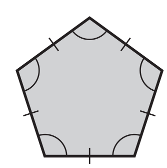
2.



3.



4.



12.6 Rotations and Symmetry

Goal: Rotate figures and identify rotational symmetry.

Vocabulary

Rotation:

Center of rotation:

Angle of rotation:

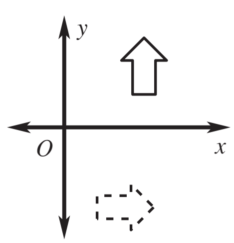
Rotational symmetry:

Example 1 ***Identifying Rotations***

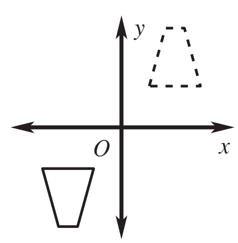
In a rotation, a figure and its image are congruent.

Tell whether the transformation is a rotation about the origin. If so, give the angle and direction of the rotation.

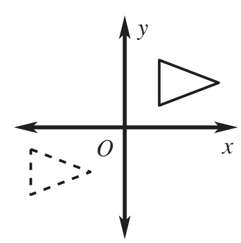
a.



b.



c.



Solution

a.

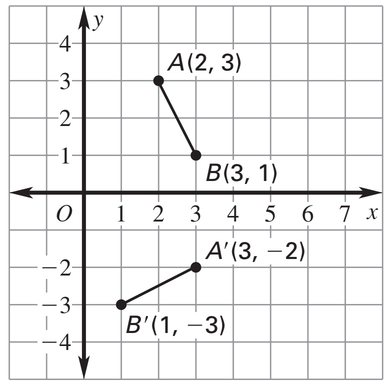
b.

c.

90° Rotations

In this lesson, all rotations in the coordinate plane are centered at the origin. You can use coordinate notation to describe a 90° rotation of a figure about the origin.

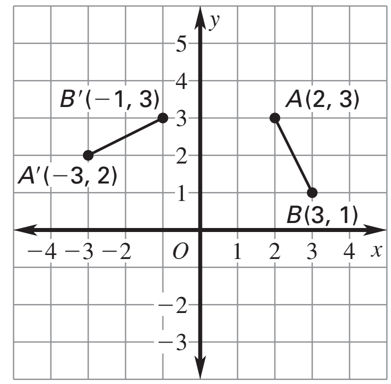
90° clockwise rotation



Switch the coordinates, then multiply the new *y*-coordinate by −1.

$(x, y) \rightarrow$ ______

90° counterclockwise rotation



Switch the coordinates, then multiply the new *x*-coordinate by −1.

$(x, y) \rightarrow$ ______

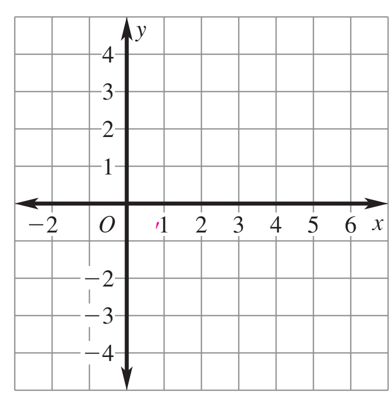
Example 2 ***Rotating a Triangle***

Draw $\triangle ABC$ with vertices $A(1, 1)$, $B(3, 4)$, and $C(4, 0)$. Then find the coordinates of the vertices of the image after a 90° clockwise rotation, and draw the image.

Solution

First draw $\triangle ABC$. Then, to rotate $\triangle ABC$ 90° clockwise, switch the coordinates and multiply the new *y*-coordinate by −1.

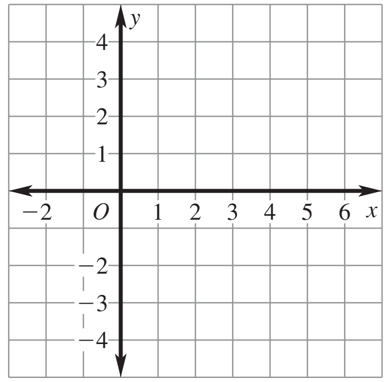
Original		**Image**
(x, y)	$\rightarrow$	______
$A(1, 1)$	$\rightarrow$	A' ______
$B(3, 4)$	$\rightarrow$	B' ______
$C(4, 0)$	$\rightarrow$	C' ______



Finally, draw $\triangle A'B'C'$.

Checkpoint

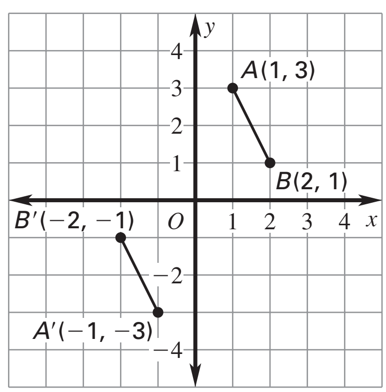
1. Draw $\triangle ABC$ with vertices $A(1, -1)$, $B(3, -1)$, and $C(4, -4)$. Then find the coordinates of the vertices of the image after a 90° counterclockwise rotation, and draw the image.



180° Rotations

To rotate a point 180° about the origin, multiply each coordinate by −1. The image is the same whether you rotate the figure ______ or ______.

$(x, y) \rightarrow$ ______



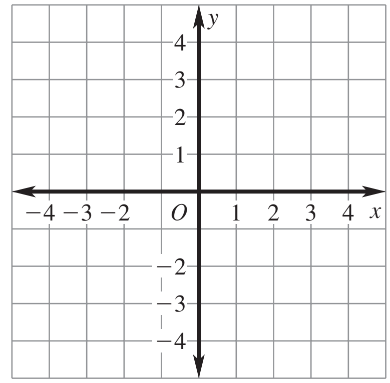
Example 3 *Rotating a Triangle*

Draw $\triangle MNP$ with vertices $M(-4, -4)$, $N(-3, -1)$, and $P(-1, -2)$. Then find the coordinates of the vertices of the image after a 180° rotation, and draw the image.

Solution

First draw $\triangle MNP$. Then, to rotate $\triangle MNP$ 180°, multiply the coordinates by -1.

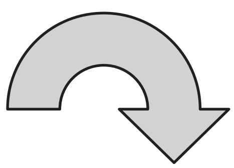
Original		Image
(x, y)	$\rightarrow$	$\boxed{}$
$M(-4, -4)$	$\rightarrow$	M' $\boxed{}$
$N(-3, -1)$	$\rightarrow$	N' $\boxed{}$
$P(-1, -2)$	$\rightarrow$	P' $\boxed{}$

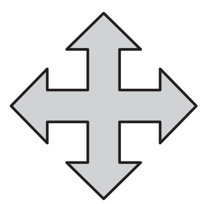


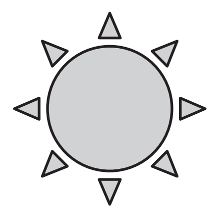
Finally, draw $\triangle M'N'P'$.

Example 4 *Identifying Rotational Symmetry*

Tell whether the figure has rotational symmetry. If so, give the angle and direction of rotation.

a. 

b. 

c. 

Solution

a. The figure $\boxed{}$ rotational symmetry.

b. The figure $\boxed{}$ rotational symmetry.

c. The figure $\boxed{}$ $\boxed{}$ rotational symmetry.

12.7 Dilations

Goal: Dilate figures in a coordinate plane.

Vocabulary

Dilation:

Center of dilation:

Scale factor:

In a dilation, a figure and its image are similar. Regardless of the value of k ($k > 0$), a point and its image after a dilation are in the same quadrant(s).

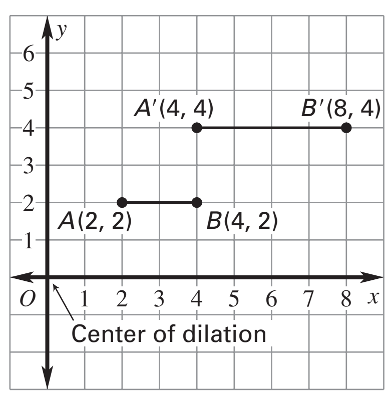
Dilation

In this lesson, the origin of the coordinate plane is the center of dilation.

In the diagram, $\overline{A'B'}$ is the image of $\overline{AB}$ after a dilation. Because $\frac{A'B'}{AB} = 2$, the scale factor is ____. You can describe a dilation with respect to the origin using the notation

$(x, y) \rightarrow (kx, ky)$

where k is the ____.



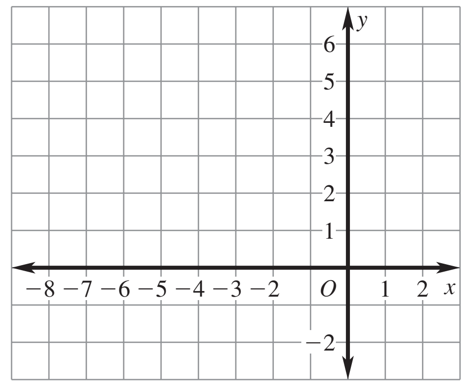
Example 1 *Dilating a Quadrilateral*

Draw quadrilateral with vertices $A(-4, 1)$, $B(1, 3)$, $C(1, -1)$, and $D(-3, -1)$. Then find the coordinates of the vertices of the image after a dilation having a scale factor of 2, and draw the image.

Solution

First draw quadrilateral *ABCD*. Then, to dilate *ABCD*, multiply the *x*- and *y*-coordinates of each vertex by ______.

Original		Image
(x, y)	$\rightarrow$	______
$A(-4, 1)$	$\rightarrow$	A' ______
$B(1, 3)$	$\rightarrow$	B' ______
$C(1, -1)$	$\rightarrow$	C' ______
$D(-3, -1)$	$\rightarrow$	D' ______

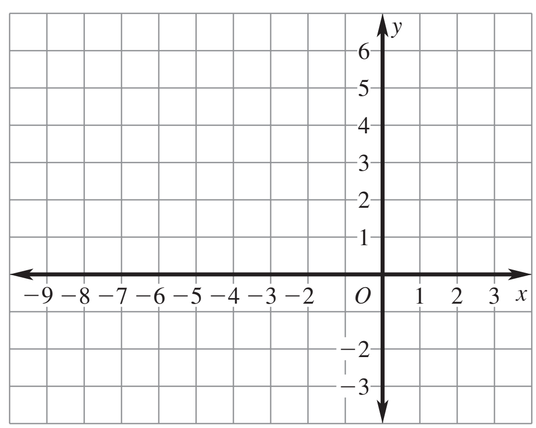


Finally, draw quadrilateral $A'B'C'D'$.

Notice in Example 1 that when $k > 1$, the new figure is an enlargement of the original figure.

Checkpoint

1. Draw $\triangle DEF$ with vertices $D(-3, 2)$, $E(1, 2)$, and $F(1, -1)$. Then find the coordinates of the vertices of the image after a dilation having a scale factor of 3, and draw the image.



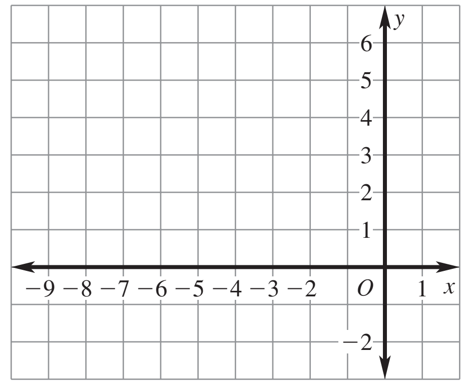
Example 2 *Using a Scale Factor Less than 1*

Draw $\triangle PQR$ with vertices $P(-8, 4)$, $Q(-6, 6)$, and $R(-4, -2)$. Then find the coordinates of the vertices of the image after a dilation having a scale factor of 0.5, and draw the image.

Solution

Draw $\triangle PQR$. Then, to dilate $\triangle PQR$, multiply the *x*- and *y*-coordinates of each vertex by $\boxed{}$.

Original		Image
(x, y)	$\rightarrow$	$\boxed{}$
$P(-8, 4)$	$\rightarrow$	$P'\boxed{}$
$Q(-6, 6)$	$\rightarrow$	$Q'\boxed{}$
$R(-4, -2)$	$\rightarrow$	$R'\boxed{}$

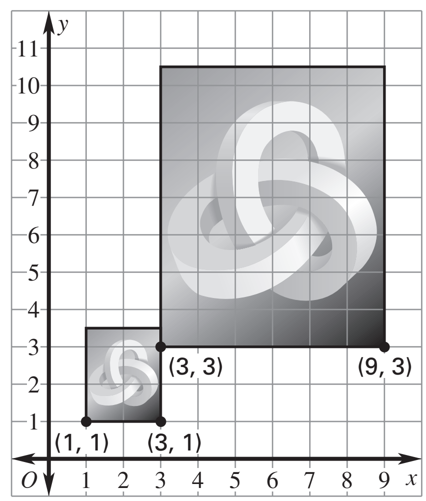


Finally, draw $\triangle P'Q'R'$.

Notice in Example 2 that when $k < 1$, the new figure is a reduction of the original figure.

Example 3 *Finding a Scale Factor*

Computer Graphics An artist uses a computer program to enlarge a design, as shown. What is the scale factor of the dilation?



Solution

The width of the original design is $\boxed{} = \boxed{}$ units. The width of the image is $\boxed{} = \boxed{}$ units. So, the scale factor is $\dfrac{\boxed{}\text{ units}}{\boxed{}\text{ units}}$, or $\boxed{}$.

Checkpoint

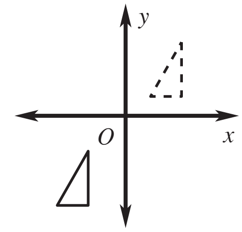
2. Given $\overline{CD}$ with endpoints $C(6, -9)$ and $D(-3, 1)$, let $\overline{C'D'}$ with endpoints $C'(2, -3)$ and $D'\left(-1, \frac{1}{3}\right)$ be the image of $\overline{CD}$ after a dilation. Find the scale factor.

$\frac{1}{3}$

Summary
Transformations in a Coordinate Plane

Translations

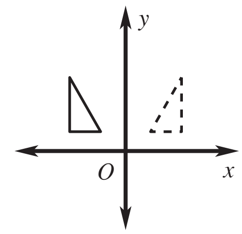
In a translation, each point of a figure is moved the ____ in the ____.



$(x, y) \rightarrow$ ____

Reflections

In a reflection, a figure is ____ over a line.

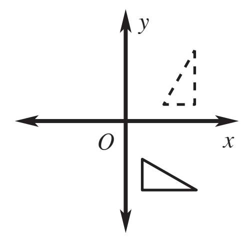


Reflection in *x*-axis: $(x, y) \rightarrow$ ____

Reflection in *y*-axis (shown): $(x, y) \rightarrow$ ____

Rotations

In a rotation, a figure is turned about the origin through a given ____ and ____.



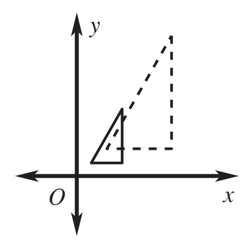
90° clockwise rotation (shown): $(x, y) \rightarrow$ ____

90° counterclockwise rotation: $(x, y) \rightarrow$ ____

180° rotation: $(x, y) \rightarrow$ ____

Dilations

In a dilation, a figure ____ or ____ with respect to the origin.



$(x, y) \rightarrow$ ____, where k is the ____.

12 Words to Review

Give an example of the vocabulary word.

Complementary angles

Supplementary angles

Vertical angles

Transversal

Corresponding angles

Alternate interior angles

Alternate exterior angles

Interior angle

Exterior angle

Transformation

Image

Translation

Tessellation

Reflection, line of reflection

Line symmetry, line of symmetry

Rotational symmetry

Rotation, center of rotation, angle of rotation

Dilation, center of dilation, scale factor

Review your notes and Chapter 12 by using the Chapter Review on pages 728–731 of your textbook.