9.1 Square Roots

Goal: Find and approximate square roots of numbers.

Vocabulary

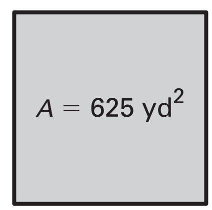
Square root:

Perfect square:

Radical expression:

Example 1 ***Finding a Square Root***

Playground A community is building a playground on a square plot of land with an area of 625 square yards. What is the length of each side of the plot of land?



Solution

The plot of land is a square with an area of 625 square yards, so the length of each side of the plot of land is the ________.

Because length cannot be negative, it doesn't make sense to find the negative square root.

$\sqrt{625} = \square$ because $\square = 625$.

Answer: The length of each side of the plot of land is ________.

Checkpoint **Find the square roots of the number.**

1. 9	**2.** 49	**3.** 169	**4.** 196

Example 2 *Approximating a Square Root*

Approximate $\sqrt{28}$ to the nearest integer.

The perfect square closest to, but less than, 28 is $\boxed{}$. The perfect square closest to, but greater than, 28 is $\boxed{}$. So, 28 is between $\boxed{}$ and $\boxed{}$. This statement can be expressed by the *compound inequality* $\boxed{} < 28 < \boxed{}$.

$\boxed{} < 28 < \boxed{}$	Identify perfect squares closest to 28.
$\boxed{} < \sqrt{28} < \boxed{}$	Take positive square root of each number.
$\boxed{} < \sqrt{28} < \boxed{}$	Evaluate square root of each perfect square.

Answer: Because 28 is closer to $\boxed{}$ than to $\boxed{}$, $\sqrt{28}$ is closer to $\boxed{}$ than to $\boxed{}$. So, to the nearest integer, $\sqrt{28} \approx \boxed{}$.

✔ ***Checkpoint*** **Approximate the square root to the nearest integer.**

5. $\sqrt{46}$	**6.** $-\sqrt{125}$	**7.** $\sqrt{68.9}$	**8.** $-\sqrt{87.5}$

Example 3 *Using a Calculator*

Use a calculator to approximate $\sqrt{636}$. Round to the nearest tenth.

Keystrokes	**Display**	**Answer**
2nd [$\sqrt{}$] 636) =	$\boxed{}$	$\boxed{}$

Checkpoint Use a calculator to approximate the square root. Round to the nearest tenth.

9. $\sqrt{6}$	10. $-\sqrt{104}$	11. $-\sqrt{819}$	12. $\sqrt{1874}$

Example 4 *Evaluating a Radical Expression*

Evaluate $5\sqrt{a^2 - b}$ when $a = 6$ and $b = 27$.

$5\sqrt{a^2 - b} = \boxed{}$ Substitute for a and for b.

$= \boxed{}$ Evaluate expression inside radical symbol.

$= \boxed{}$ Evaluate square root.

$= \boxed{}$ Multiply.

Checkpoint Evaluate the expression when $a = 16$ and $b = 9$.

13. $-\sqrt{a + b}$	14. $\sqrt{b^2 - 2a}$	15. $2\sqrt{ab}$

9.2 Simplifying Square Roots

Goal: Simplify radical expressions.

Vocabulary

Simplest form:

Product Property of Square Roots

Algebra

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a \geq 0$ and $b \geq 0$

Numbers

$\sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7} = 3\sqrt{7}$

Example 1 *Simplifying a Radical Expression*

$\sqrt{150} = \boxed{}$ Factor using greatest perfect square factor.

$= \boxed{}$ Product property of square roots

$= \boxed{}$ Simplify.

Example 2 *Simplifying a Variable Expression*

$\sqrt{18t^2} = \boxed{}$ Factor using greatest perfect square factor.

$= \boxed{}$ Product property of square roots

$= \boxed{}$ Simplify.

$= \boxed{}$ Commutative property

Checkpoint Simplify the expression.

1. $\sqrt{75}$	2. $\sqrt{80}$	3. $\sqrt{24r^2}$	4. $\sqrt{56m^2}$

Quotient Property of Square Roots

Algebra

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Numbers

$\sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{\sqrt{4}} = \frac{\sqrt{11}}{2}$

Example 3 *Simplifying a Radical Expression*

$\sqrt{\frac{17}{25}} = \boxed{}$ Quotient property of square roots

$= \boxed{}$ Simplify.

Checkpoint Simplify the expression.

5. $\sqrt{\frac{5}{9}}$	6. $\sqrt{\frac{21}{64}}$	7. $\sqrt{\frac{16z}{49}}$	8. $\sqrt{\frac{32y^2}{81}}$

Example 4 ***Using Radical Expressions***

The expression $\sqrt{\dfrac{h}{16}}$ gives the time (in seconds) it takes an object to fall h feet.

a. Write the expression in simplest form.

b. Find the length of time it takes for an object to fall 120 feet. Give your answer to the nearest tenth of a second.

Solution

a. $\sqrt{\dfrac{h}{16}} = \boxed{}$ Quotient property of square roots

$= \boxed{}$ Simplify.

Answer: In simplest form, $\sqrt{\dfrac{h}{16}} = \boxed{}$.

b. $\dfrac{\sqrt{h}}{\boxed{}} = \boxed{}$ Substitute 120 for h.

$\approx \boxed{}$ Approximate using a calculator.

Answer: The length of time it takes for an object to fall 120 feet is about $\boxed{}$.

Checkpoint

9. Find the length of time it takes for an object to fall 155 feet. Give your answer to the nearest tenth of a second.

Focus On Operations

Use after Lesson 9.2

Performing Operations on Square Roots

Goal: Perform operations on square roots.

Example 1 *Multiplying and Dividing Radical Expressions*

a. $\sqrt{12} \cdot \sqrt{2} = \sqrt{12 \cdot 2}$ — Product property of square roots

$= \boxed{}$ — Multiply.

$= \boxed{} \cdot \boxed{}$ — Product property of square roots

$= \boxed{}$ — Simplify.

b. $\dfrac{\sqrt{18}}{\sqrt{10}} = \sqrt{\dfrac{\boxed{}}{\boxed{}}}$ — Quotient property of square roots

$= \sqrt{\dfrac{\boxed{}}{\boxed{}}}$ — Simplify.

$= \dfrac{\sqrt{\boxed{}}}{\sqrt{\boxed{}}}$ — Quotient property of square roots

$= \dfrac{\boxed{}}{\sqrt{\boxed{}}}$ — Simplify.

$= \dfrac{\boxed{}}{\sqrt{\boxed{}}} \cdot \dfrac{\sqrt{\boxed{}}}{\sqrt{\boxed{}}}$ — Rationalize denominator.

$= \dfrac{\boxed{}\sqrt{\boxed{}}}{\boxed{}}$ — Multiply and simplify.

The process of rewriting a fraction so that its denominator is a rational number is called *rationalizing the denominator*.

Example 2 *Adding and Subtracting Radical Expressions*

a. $5\sqrt{6} + 3\sqrt{2} - 4\sqrt{6} = \boxed{} + 3\sqrt{2}$ — Commutative property

$= (\boxed{})\sqrt{\boxed{}} + 3\sqrt{2}$ — Distributive property

$= \boxed{} + 3\sqrt{2}$ — Simplify.

b. $9\sqrt{3} + \sqrt{12} = 9\sqrt{3} + \sqrt{\boxed{} \cdot \boxed{}}$ — Factor using perfect square factor.

$= 9\sqrt{3} + \boxed{} \cdot \boxed{}$ — Product property of square roots

$= 9\sqrt{3} + \boxed{}$ — Simplify.

$= (\boxed{})\sqrt{\boxed{}}$ — Distributive property

$= \boxed{}$ — Simplify.

Checkpoint Simplify the expression.

1. $\sqrt{12} \cdot \sqrt{6}$	2. $\dfrac{\sqrt{48}}{\sqrt{21}}$
3. $5\sqrt{8} + 6\sqrt{2}$	4. $8\sqrt{3} - 3\sqrt{11} - \sqrt{3}$

Example 3 *Using Radical Expressions*

Circular plate The expression $\sqrt{\dfrac{A}{3.14}}$ gives the radius (in centimeters) of a circular plate with area A (in square centimeters). About how much longer is the radius of a circular plate with an area of 628 square centimeters than the radius of a circular plate with an area of 157 square centimeters?

1. Find the radius when $A = 628$.

$\sqrt{\dfrac{A}{3.14}} = \sqrt{\dfrac{\square}{\square}}$ Substitute $\square$ for A.

$= \square$ Divide.

$= \square \cdot \sqrt{2}$ Product property of square roots

$= \square$ Simplify.

2. Find the radius when $A = 157$.

$\sqrt{\dfrac{A}{3.14}} = \sqrt{\dfrac{\square}{\square}}$ Substitute $\square$ for A.

$= \square$ Divide.

$= \square \cdot \sqrt{2}$ Product property of square roots

$= \square$ Simplify.

3. Find the difference of the lengths.

$10\sqrt{2} - 5\sqrt{2} = (\square)\square$ Distributive property

$= \square \approx \square \cdot 1.4 \approx \square$ Simplify and approximate.

Answer about $\square$ centimeters

9.3 The Pythagorean Theorem

Goal: Use the Pythagorean theorem to solve problems.

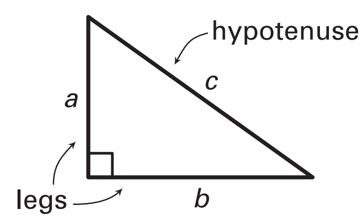
Vocabulary

Hypotenuse:

Legs:

Pythagorean Theorem

Words For any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.



Algebra $a^2 + b^2 = c^2$

Example 1 *Finding the Length of a Hypotenuse*

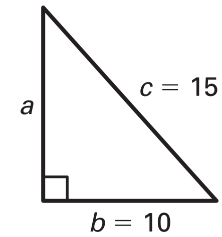
A building's access ramp has a horizontal distance of 24 feet and a vertical distance of 2 feet. Find the length of the ramp to the nearest tenth of a foot.

$a^2 + b^2 = c^2$	Pythagorean theorem
$\boxed{}^2 + \boxed{}^2 = c^2$	Substitute for *a* and for *b*.
$\boxed{} = c^2$	Evaluate powers and add.
$\boxed{} = c$	Take positive square root of each side.
$\boxed{} \approx c$	Simplify.

Answer: The length of the ramp is about $\boxed{}$ feet.

Example 2 *Finding the Length of a Leg*

Find the unknown length *a* in simplest form.

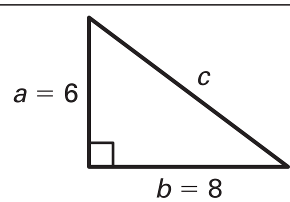


$a^2 + b^2 = c^2$	Pythagorean theorem
$a^2 + \boxed{}^2 = \boxed{}^2$	Substitute.
$a^2 + \boxed{} = \boxed{}$	Evaluate powers.
$a^2 = \boxed{}$	Subtract $\boxed{}$ from each side.
$a = \boxed{}$	Take positive square root of each side.
$a = \boxed{}$	Simplify.

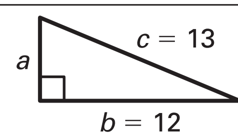
Answer: The unknown length *a* is $\boxed{}$ units.

Checkpoint **Find the unknown length. Write your answer in simplest form.**

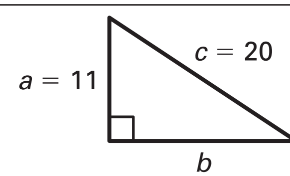
1.



2.



3.



Converse of the Pythagorean Theorem

The Pythagorean theorem can be written in "if-then" form.

Theorem: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

If you reverse the two parts of the statement, the new statement is called the *converse* of the Pythagorean theorem.

Converse: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Although not all converses of true statements are true, the converse of the Pythagorean theorem is true.

Example 3 *Identifying Right Triangles*

Determine whether the triangle with the given side lengths is a right triangle.

a. $a = 8, b = 9, c = 12$

b. $a = 7, b = 24, c = 25$

Solution

a. $a^2 + b^2 = c^2$

$\boxed{}^2 + \boxed{}^2 \stackrel{?}{=} \boxed{}^2$

$\boxed{} + \boxed{} \stackrel{?}{=} \boxed{}$

$\boxed{}\ \boxed{}\ \boxed{}$

Answer: $\boxed{}$

b. $a^2 + b^2 = c^2$

$\boxed{}^2 + \boxed{}^2 \stackrel{?}{=} \boxed{}^2$

$\boxed{} + \boxed{} \stackrel{?}{=} \boxed{}$

$\boxed{}\ \boxed{}\ \boxed{}$

Answer: $\boxed{}$

✔ ***Checkpoint*** **Determine whether the triangle with the given side lengths is a right triangle.**

4. $a = 12, b = 9, c = 15$	**5.** $a = 10, b = 25, c = 27$

Focus On Geometry

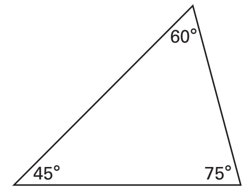
Use after Lesson 9.3

Angles of a Triangle

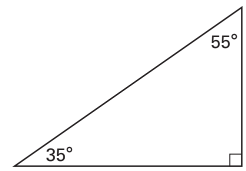
Goal: Determine the sum of the measures of the angles of a triangle.

Vocabulary

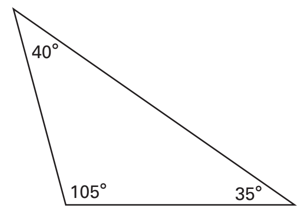
Acute triangle



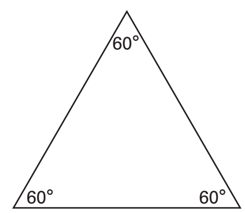
Right triangle



Obtuse triangle

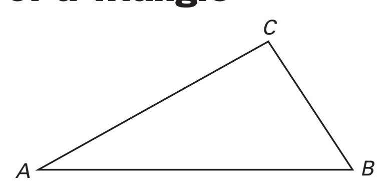


Equiangular triangle



Sum of the Measures of the Angles of a Triangle

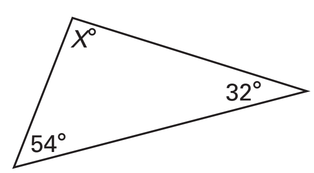
The sum of the measures of the interior angles of a triangle is 180°.



$$m\angle A + m\angle B + m\angle C = 180°$$

Example 1 *Finding an Angle Measure in a Triangle*

Find the value of x.



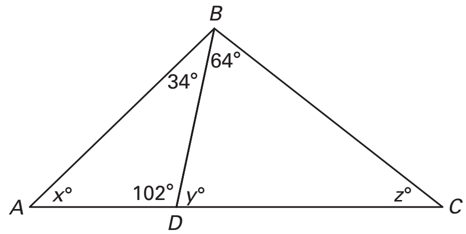
$54° + \square + x° = \square$ Sum of angle measures is 180°.

$\square + x = \square$ Add.

$x = \square$ Subtract $\square$ from each side.

Example 2 *Classifying a Triangle by its Angle Measures*

Find the values of x, y, and z. Then classify all triangles in the diagram by their angle measures.



1. Write and solve an equation to find the value of x.

$x° + 34° + \boxed{} = \boxed{}$ Sum of angle measures is 180°.

$x + \boxed{} = \boxed{}$ Add.

$x = \boxed{}$ Subtract $\boxed{}$ from each side.

2. Write and solve an equation to find the value of y.

$y° + \boxed{} = \boxed{}$ The measure of a straight angle is 180°.

$y = \boxed{}$ Subtract $\boxed{}$ from each side.

3. Write and solve an equation to find the value of z.

$z° + 78° + \boxed{} = \boxed{}$ Sum of angle measures is 180°.

$z + \boxed{} = \boxed{}$ Add.

$z = \boxed{}$ Subtract $\boxed{}$ from each side.

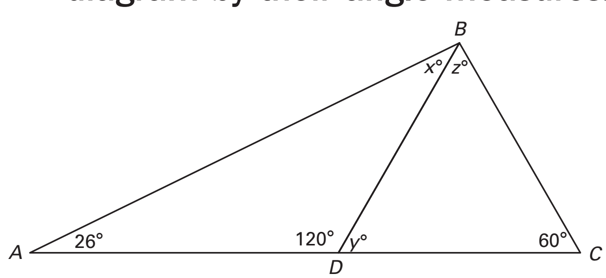
4. $\triangle ABD$ has $\boxed{}$, so it is a(n) $\boxed{}$ triangle.

$\triangle BCD$ has $\boxed{}$, so it is an $\boxed{}$ triangle.

The measure of $\angle B$ is $34° + \boxed{} = \boxed{}$, so $\triangle ABC$ has $\boxed{}$ and is a(n) $\boxed{}$ triangle.

Checkpoint **Complete the following exercise.**

1. Find the values of x, y, and z. Then classify all triangles in the diagram by their angle measures.



9.4 Real Numbers

Goal: Compare and order real numbers.

Vocabulary

Irrational number: ______

Real numbers: ______

Example 1 *Classifying Real Numbers*

	Number	Decimal Form	Decimal Type	Type
a.	$\frac{7}{10}$	$\frac{7}{10}$ = ______	______	______
b.	$\frac{2}{9}$	$\frac{2}{9}$ = ______ = ______	______	______
c.	$\sqrt{5}$	$\sqrt{5}$ = ______	______ ______	______

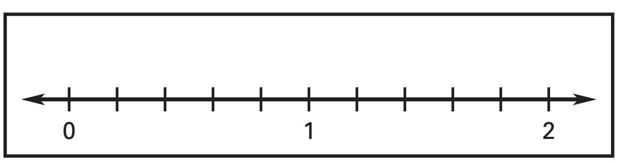
The square root of any whole number that is not a perfect square is irrational.

Example 2 *Comparing Real Numbers*

Copy and complete $\sqrt{3}$ $\underline{\ ?\ }$ $\frac{6}{5}$ using <, >, or =.

Solution

Graph $\sqrt{3}$ and $\frac{6}{5}$ on a number line.



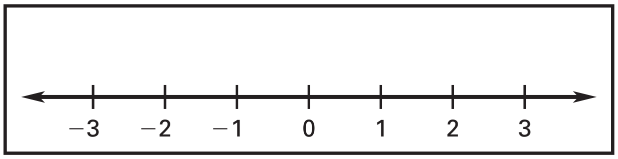
$\sqrt{3}$ is to the ______ of $\frac{6}{5}$.

Answer: $\sqrt{3}$ ______ $\frac{6}{5}$

Example 3 *Ordering Real Numbers*

Use a number line to order the numbers $\frac{\sqrt{6}}{2}$, -2.2, $\frac{5}{2}$, and $-2\sqrt{2}$ from least to greatest.

Graph the numbers on a number line and read them from left to right.



Answer: From least to greatest, the numbers are

______________________.

✔ ***Checkpoint*** **Tell whether the number is *rational* or *irrational*.**

1. $\frac{8}{11}$	**2.** $\sqrt{7}$	**3.** $\sqrt{49}$	**4.** $\sqrt{\frac{2}{5}}$

Copy and complete the statement using $<$, $>$, or $=$.

5. $-\frac{3}{2}$ $\underline{\ ?\ }$ $-\sqrt{3}$	**6.** $\sqrt{10}$ $\underline{\ ?\ }$ 3.5

✔ ***Checkpoint*** **Use a number line to order the numbers from least to greatest.**

7. $5.9, 3\sqrt{5}, \frac{27}{5}, \sqrt{35}$

8. $-\sqrt{21}, -4.6, -\frac{9}{2}, -2\sqrt{5}$

Example 4 *Using Irrational Numbers*

Speed After an accident, a police officer finds that the length of a car's skid marks is 98 feet. The car's speed s (in miles per hour) and the length ℓ (in feet) of the skid marks are related by $s = \sqrt{27\ell}$. Find the car's speed to the nearest tenth of a mile per hour.

Solution

$s = \sqrt{27\ell}$	Write formula.
$= \sqrt{27 \cdot \square}$	Substitute value.
$= \sqrt{\square}$	Multiply.
$\approx \square$	Approximate using a calculator.

Answer: The car's speed was about $\square$ miles per hour.

Focus On Operations

Use after Lesson 9.4

Approximating Irrational Numbers

Goal: Use rational approximations to approximate and compare irrational numbers.

Vocabulary

Square root: $\square$

Perfect square: $\square$

Irrational number: $\square$

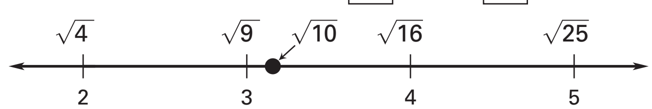
Example 1 *Approximating an Irrational Number*

Approximate $\sqrt{10}$ to the nearest tenth.

Create a table of numbers whose squares are close to 10.

Number	2	3	4	5
Square	$\square$	$\square$	$\square$	$\square$

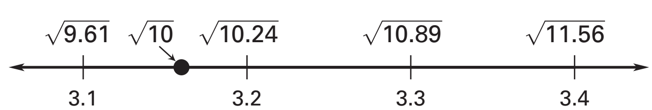
The table shows that 10 is between the perfect squares $\square$ and $\square$. So, $\sqrt{10}$ is between $\square$ and $\square$.



Create a table of numbers between $\boxed{3}$ and $\boxed{4}$ whose squares are close to 10.

Number	3.1	3.2	3.3	3.4
Square	$\square$	$\square$	$\square$	$\square$

The average of $\square$ and $\square$ is $\square$ and $(\square)^2 = \square$. Because $\square$ $\square$ 10, $\sqrt{10}$ is closer to $\square$ than to $\square$.



Answer So, $\sqrt{10} \approx \square$.

Checkpoint Approximate to the nearest tenth.

1. $\sqrt{32}$	2. $-\sqrt{23}$

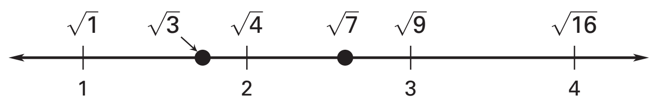
Example 2 *Comparing Irrational Numbers*

Copy and complete $3\sqrt{3}\ \underline{?}\ 2\sqrt{7}$ using $<$, $>$, or $=$.

Consider the numbers inside the radical symbols first. Create a table of numbers whose squares are close to 3 and 7.

Number	1	2	3	4
Square	$\square$	$\square$	$\square$	$\square$

The table shows that 3 is between the perfect squares $\square$ and $\square$, and 7 is between the perfect squares $\square$ and $\square$.



Create a table of numbers between $\square$ and $\square$ whose squares are close to 3, and another table of numbers between $\square$ and $\square$ whose squares are close to 7.

Number	1.7	1.8	1.9
Square	$\square$	$\square$	$\square$

Number	2.6	2.7	2.8
Square	$\square$	$\square$	$\square$

The average of 1.7 and 1.8 is $\square$, and $(\square)^2 = \square$. Because 3 $\square$ $\square$, $\sqrt{3}$ is closer to $\square$ than to $\square$. The average of 2.6 and 2.7 is $\square$, and $(\square)^2 = \square$. Because 7 $\square$ $\square$, $\sqrt{7}$ closer to $\square$ than to $\square$.

So, $3\sqrt{3} \approx 3(\square) = \square$ and $2\sqrt{7} \approx 2(\square) = \square$.

Answer $3\sqrt{3}\ \square\ 2\sqrt{7}$

Checkpoint Complete the statement using $<$, $>$, or $=$.

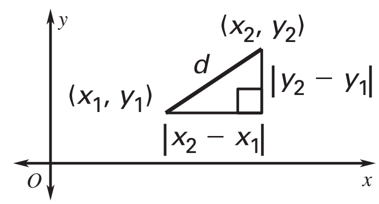
3. $2\sqrt{28}\ \square\ \sqrt{85}$	4. $\sqrt{110}\ \square\ 3\sqrt{8}$

9.5 Distance and Midpoint Formulas

Goal: Use the distance, midpoint, and slope formulas.

The Distance Formula

Words The distance between two points in a coordinate plane is equal to the square root of the sum of the horizontal change squared and the vertical change squared.



Algebra $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1 *Finding the Distance Between Two Points*

Find the distance between the points $M(2, 3)$ and $N(1, 5)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$= \sqrt{(1 - \boxed{})^2 + (5 - \boxed{})^2}$	Substitute.
$= \sqrt{(\boxed{})^2 + \boxed{}^2}$	Subtract.
$= \sqrt{\boxed{} + \boxed{}}$	Evaluate powers.
$= \boxed{}$	Add.

Answer: The distance between the points is $\boxed{}$ units.

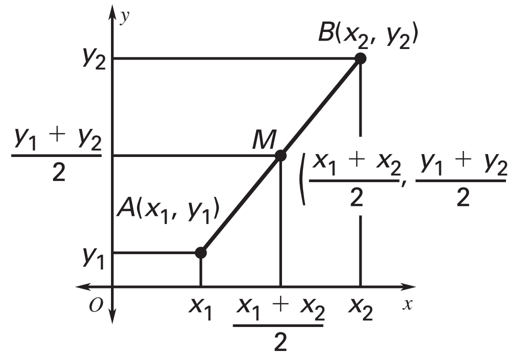
✔ ***Checkpoint*** **Find the distance between the points. Write your answer in simplest form.**

1. (2, −5), (−3, 7)	**2.** (2, −2), (0, 4)

The Midpoint Formula

Words The coordinates of the midpoint of a segment are the average of the endpoints' x-coordinates and the average of the endpoints' y-coordinates.

Algebra $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



Example 2 ***Finding a Midpoint***

Find the midpoint M of the segment with endpoints (2, 5) and (−6, −3).

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint formula

$= \left(\frac{\boxed{} + \boxed{}}{2}, \frac{\boxed{} + \boxed{}}{2}\right)$ Substitute values.

$= \boxed{}$ Simplify.

Checkpoint **Find the midpoint of the segment with the given endpoints.**

3. (2, −5), (−3, 7)	**4.** (3, −4), (7, 2)

Slope

If points $A(x_1, y_1)$ and $B(x_2, y_2)$ do not lie on a vertical line, you can use coordinate notation to write a formula for the slope of the line through A and B.

$$\text{slope} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 3 ***Finding Slope***

Find the slope of the line through (2, −1) and (−1, 2).

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}}$ Substitute values.

$= \frac{\boxed{}}{\boxed{}} = \boxed{}$ Simplify.

✔ ***Checkpoint*** **Find the slope of the line through the given points.**

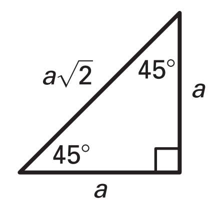
5. (2, −6), (−3, 9)	**6.** (6, −2), (−4, 7)

9.6 Special Right Triangles

Goal: Use special right triangles to solve problems.

45°-45°-90° Triangle

Words In a 45°-45°-90° triangle, the length of the hypotenuse is the product of the length of a leg and $\sqrt{2}$.

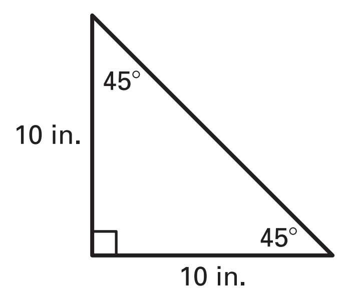


Algebra hypotenuse = leg • $\sqrt{2}$

$= a\sqrt{2}$

Example 1 *Using a 45°-45°-90° Triangle*

A 45°-45°-90° triangle used in mechanical drawing has 10-inch legs. Find the length of the hypotenuse to the nearest tenth of an inch.



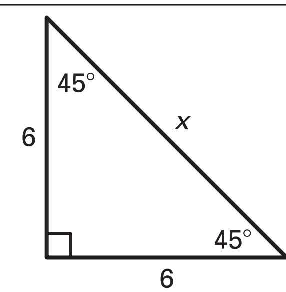
Solution

hypotenuse = leg • $\sqrt{2}$		Rule for 45°-45°-90° triangle
= ☐ • $\sqrt{2}$		Substitute.
≈ ☐		Use a calculator.

Answer: The length of the triangle's hypotenuse is about ☐ inches.

Checkpoint

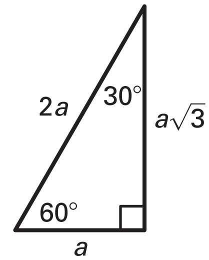
1. Find the unknown length *x*. Write your answer in simplest form.



In a 30°-60°-90° triangle, the shorter leg is opposite the 30° angle, and the longer leg is opposite the 60° angle.

30°-60°-90° Triangle

Words In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is the product of the length of the shorter leg and $\sqrt{3}$.

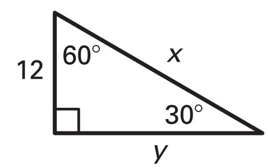


Algebra hypotenuse = 2 • shorter leg = $2a$

longer leg = shorter leg • $\sqrt{3} = a\sqrt{3}$

Example 2 *Using a 30°-60°-90° Triangle*

Find the length x of the hypotenuse and the length y of the longer leg of the triangle.



The triangle is a 30°-60°-90° triangle.
The length of the shorter leg is ☐ units.

a. hypotenuse = 2 • shorter leg

$x = 2 \cdot \boxed{}$

$= \boxed{}$

Answer: The length x of the hypotenuse is ☐ units.

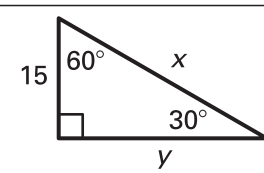
b. longer leg = shorter leg • $\sqrt{3}$

$y = \boxed{}\sqrt{3}$

Answer: The length y of the longer leg is ☐$\sqrt{3}$ units.

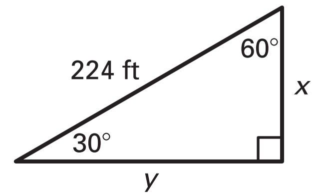
✔ ***Checkpoint***

2. Find the unknown lengths x and y. Write your answers in simplest form.



Example 3 ***Using a Special Right Triangle***

An escalator going up to the second floor in a mall is 224 feet long and makes a 30° angle with the first floor. Find, to the nearest foot, the lengths of the triangle's legs.



Solution

You need to find the length of the shorter leg first.

1. Find the length x of the shorter leg.

hypotenuse = 2 • shorter leg	Rule for 30°-60°-90° triangle
$\boxed{} = 2x$	Substitute.
$\boxed{} = x$	Divide each side by $\boxed{}$.

2. Find the length y of the longer leg.

longer leg = shorter leg • $\sqrt{3}$	Rule for 30°-60°-90° triangle
$y = \boxed{}\sqrt{3}$	Substitute.
$y \approx \boxed{}$	Use a calculator.

Answer: The length of the shorter leg is $\boxed{}$ feet and the length of the longer leg is about $\boxed{}$ feet.

9.7 The Tangent Ratio

Goal: Use tangent to find side lengths of right triangles.

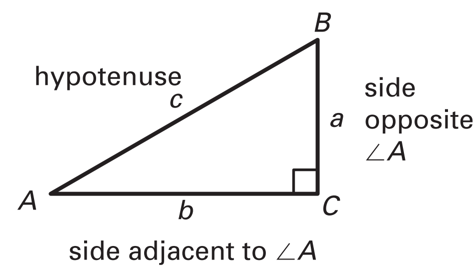
Vocabulary

Trigonometric ratio:

The Tangent Ratio

The **tangent** of an acute angle of a right triangle is the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle.

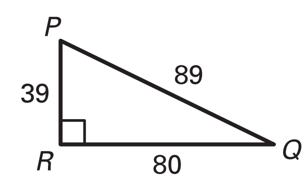
$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$



Example 1 *Finding a Tangent Ratio*

For $\triangle PQR$, find the tangent of $\angle P$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}} = \boxed{}$$



Checkpoint

1. For $\triangle PQR$ in Example 1, find the tangent of $\angle Q$.

Example 2 *Using a Calculator*

When using a calculator to find a trigonometric ratio, make sure the calculator is in degree mode. Round the result to four decimal places if necessary.

a. tan 24°

Keystrokes	Display	Answer
2nd [TRIG] ◀ ◀ = 24) =		

b. tan 55°

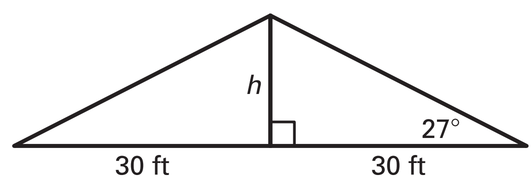
Keystrokes	Display	Answer
2nd [TRIG] ◀ ◀ = 55) =		

✔ ***Checkpoint*** **Approximate the tangent value to four decimal places.**

2. tan 5°	**3.** tan 38°	**4.** tan 72°

Example 3 *Using a Tangent Ratio*

Find the height h (in feet) of the roof to the nearest foot.



Solution

Use the tangent ratio. In the diagram, the length of the leg opposite the 27° angle is h. The length of the adjacent leg is 30 feet.

$\tan 27^\circ = \frac{\text{opposite}}{\text{adjacent}}$ Definition of tangent ratio

$\tan 27^\circ = \boxed{}$ Substitute.

$\boxed{} \approx \boxed{}$ Use a calculator to approximate tan 27°.

$\boxed{} \approx h$ Multiply each side by $\boxed{}$.

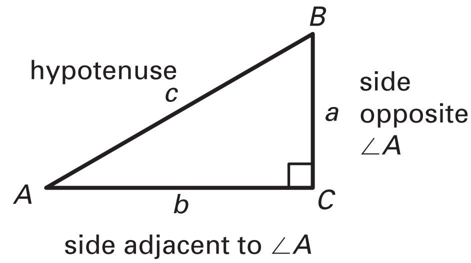
Answer: The height of the roof is about $\boxed{}$ feet.

9.8 The Sine and Cosine Ratios

Goal: Use sine and cosine to find triangle side lengths.

The Sine and Cosine Ratios

The **sine** of an acute angle of a right triangle is the ratio of the length of the side opposite the angle to the length of the hypotenuse.



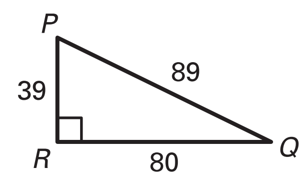
$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

The **cosine** of an acute angle of a right triangle is the ratio of the length of the angle's adjacent side to the length of the hypotenuse.

$$\cos A = \frac{\text{side adjacent } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

Example 1 ***Finding Sine and Cosine Ratios***

For $\triangle PQR$, find the sine and cosine of $\angle P$.



$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}} = \boxed{}$$

$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}} = \boxed{}$$

Checkpoint

1. For $\triangle PQR$ in Example 1, find the sine and cosine of $\angle Q$.

Example 2 *Using a Calculator*

a. sin 60°

Keystrokes	**Display**	**Answer**
2nd [TRIG] = 60) =		

b. cos 45°

Keystrokes	**Display**	**Answer**
2nd [TRIG] ▶ ▶ = 45) =		

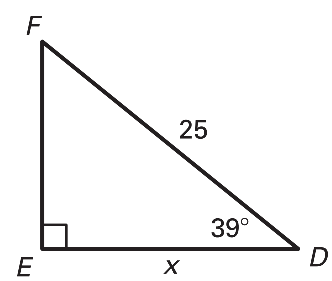
✔ ***Checkpoint*** **Approximate the sine or cosine value to four decimal places.**

2. cos 9°	**3.** cos 78°	**4.** sin 13°	**5.** sin 88°

Example 3 *Using a Cosine Ratio*

Find the value of *x* in the triangle.

In $\triangle DEF$, $\overline{DE}$ is adjacent to $\angle D$. Because you know the length of the hypotenuse, use cos *D* and the definition of the cosine ratio to find the value of *x*. Round your answer to the nearest tenth of a unit.



$\cos D = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ Definition of cosine ratio

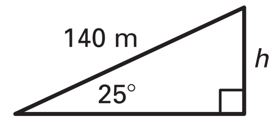
$\boxed{} = \boxed{}$ Substitute.

$\boxed{} \approx \boxed{}$ Use a calculator to approximate cos 39°.

$\boxed{} \approx x$ Multiply each side by $\boxed{}$.

Example 4 *Using a Sine Ratio*

A ski jump is 140 meters long and makes an angle of 25° with the ground. To the nearest meter, estimate the height h of the ski jump.



Solution

To estimate the height of the ski jump, find the length of the side $\boxed{}$ the 25° angle. Because you know the length of the hypotenuse, use sin 25°.

$$\sin 25^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$ Definition of sine ratio

$$\sin 25^\circ = \boxed{}$$ Substitute.

$$\boxed{} \approx \boxed{}$$ Use a calculator to approximate sin 25°.

$$\boxed{} \approx h$$ Multiply each side by $\boxed{}$.

Answer: The height of the ski jump is about $\boxed{}$ meters.

9 Words to Review

Give an example of the vocabulary word.

Square root	Perfect square

Radical expression	Simplest form of a radical expression

Hypotenuse	Leg

Pythagorean theorem	Acute triangle

Right triangle

Obtuse triangle

Equiangular triangle

Irrational number

Real number

Distance formula

Midpoint formula

Slope formula

Sine

Cosine

Tangent

Review your notes and Chapter 9 by using the Chapter Review on pages 526–529 of your textbook.