8.1 Relations and Functions

Goal: Use graphs to represent relations and functions.

Vocabulary

Relation:

Domain:

Range:

Input:

Output:

Function:

Vertical line test:

Example 1 ***Identifying the Domain and Range***

Identify the domain and range of the relation represented by the table below that shows one Norway Spruce tree's height at different ages.

Age (years), *x*	5	10	15	20	25
Height (ft), *y*	13	25	34	43	52

Solution

The relation consists of the ordered pairs _____ _____. The domain of the relation is the set of all _____, or _____. The range is the set of all _____, or _____.

Domain: _____ **Range:** _____

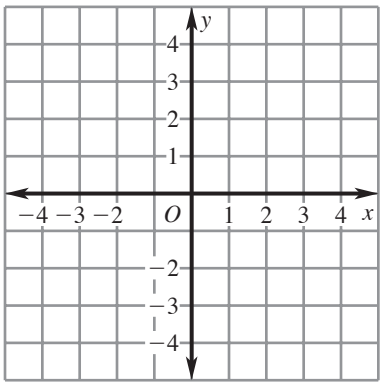
Example 2 *Representing a Relation*

Represent the relation (−3, 2), (−2, −2), (1, 1), (1, 3), (2, −3) as indicated.

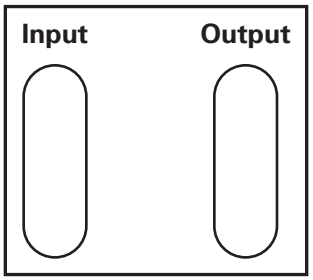
a. A graph **b.** A mapping diagram

Solution

a. Graph the ordered pairs as _____ in a coordinate plane.



b. List the inputs and the outputs in order. Draw arrows from the _____ to their _____.



Example 3 *Identifying Functions*

Tell whether the relation is a function.

a. The relation in Example 1.

b. The relation in Example 2.

Solution

a. The relation _____ a function because _____ _____. This makes sense, as a single tree can have _____ height at a given point in time.

b. The relation _____ a function because _____ _____.

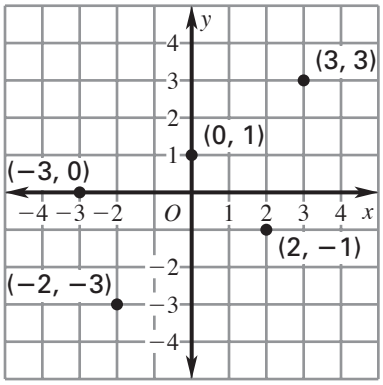
✔ ***Checkpoint*** **Identify the domain and range of the relation and tell whether the relation is a function.**

1. (−5, 2), (−3, −1), (−1, 0), (2, 3), (5, 4)	**2.** (−4, −3), (−3, 2), (0, 0), (1, −1), (2, 3), (3, 1), (3, −2)

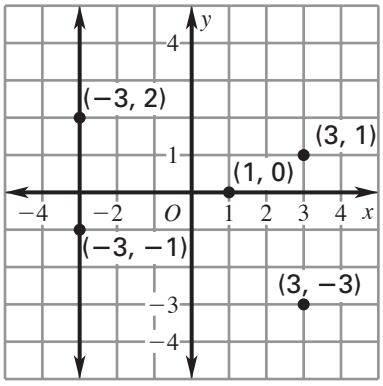
To understand why the vertical line test works, remember that a function has exactly one output for each input.

Example 4 *Using the Vertical Line Test*

a. In the graph below, no vertical line passes through more than one point. So, the relation represented by the graph _____.



b. In the graph below, the vertical line shown passes through two points. So, the relation represented by the graph _____.



8.2 Linear Equations in Two Variables

Goal: Find solutions of equations in two variables.

Vocabulary

Equation in two variables:

Solution of an equation in two variables:

Graph of an equation in two variables:

Linear equation:

Linear function:

Function form:

Example 1 ***Checking Solutions***

Tell whether (5, −1) is a solution of $x - 3y = 8$.

Solution

$x - 3y = 8$	Write original equation.
$\square - 3(\square) \stackrel{?}{=} 8$	Substitute for x and for y.
$\square \quad \square \quad 8$	Simplify.

Answer: (5, −1) $\square$ a solution of $x - 3y = 8$.

✔ ***Checkpoint*** **Tell whether the ordered pair is a solution of $2x - y = 5$.**

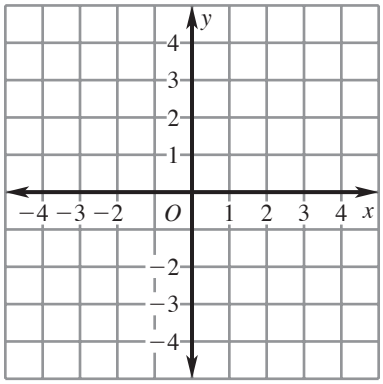
1. (0, −5)	**2.** (3, 2)	**3.** (−2, −9)

Example 2 *Graphing a Linear Equation*

Graph $y = -x + 1$.

1. Make a table of solutions.

x	−2	−1	0	1	2
y					



2. List the solutions as ordered pairs.

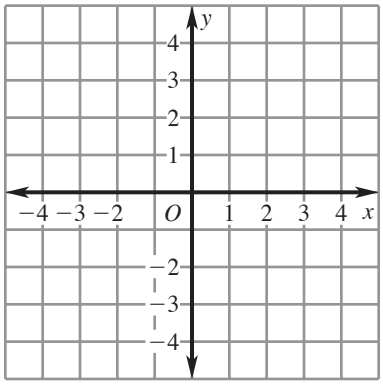
(−2,), (−1,), (0,),
(1,), (2,)

3. Graph the ordered pairs, and note that the points lie on a . Draw the , which is the graph of $y = -x + 1$.

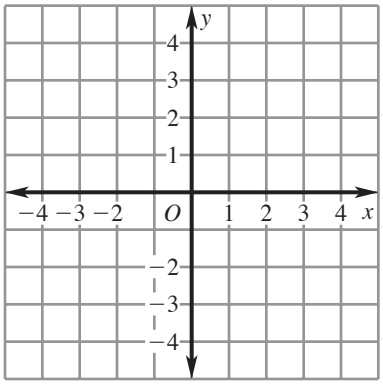
Example 3 *Graphing Horizontal and Vertical Lines*

Graph $y = -2$ and $x = 3$.

a. The graph of the equation $y = -2$ is .



b. The graph of the equation $x = 3$ is .



Example 4 *Writing an Equation in Function Form*

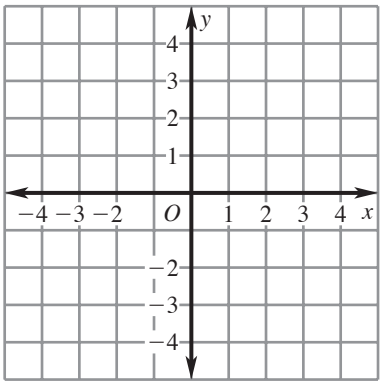
Write $3x - y = 2$ in function form. Then graph the equation.

To write the equation in function form, solve for ☐.

$3x - y = 2$	Write original equation.
☐ = ☐ + 2	Subtract ☐ from each side.
☐ = ☐ − 2	Multiply each side by ☐.

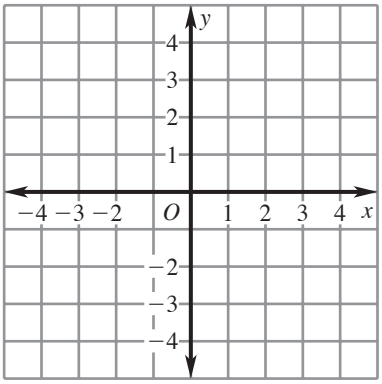
To graph the equation, use its function form to make a table of solutions. Graph the ordered pairs (x, y) from the table, and draw a line through the points.

x	−1	0	1	2
y	☐	☐	☐	☐

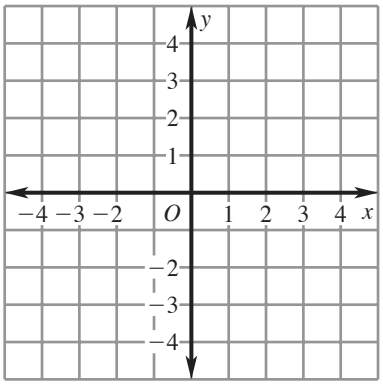


✔ *Checkpoint*

4. Graph $y = 4$ and $x = -3$. Tell whether each equation is a function.



5. Write $x - 2y = 4$ in function form. Then graph the equation.



Focus on Algebra

Use after Lesson 8.2

Linear and Nonlinear Functions

Goal: Understand that functions can be linear or nonlinear.

Vocabulary

Nonlinear function

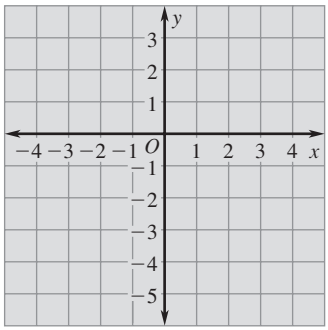
Increasing function

Decreasing function

Example 1 ***Analyzing the Graph of a Linear Function***

Graph the linear function $y = -0.5x - 1$. Is the function increasing or decreasing? Explain.

Make a table of solutions. Then graph the ordered pairs (x, y) and draw a through the points.



x	−4	−2	0	2	4
y					

Because the graph from left to right, the function is .

Example 2 *Analyzing the Graph of a Nonlinear Function*

Graph the nonlinear function $y = x^3 - 2$. Is the function increasing or decreasing? Explain.

Make a table of solutions. Then graph the ordered pairs (x, y) and draw a _____ through the points.

x	−2	−1	0	1	2
y					

Because the graph _____ from left to right, the function is _____.

Example 3 *Graphing a Function that is Described Verbally*

Driveway Consider a driveway that is 15 feet wide. Graph the perimeter of the rectangular driveway as a function of its length l. Is the function linear or nonlinear? Increasing or decreasing? Explain.

Solution

1. Write a function using the formula for _____.

$P = 2w + 2l$	Formula for perimeter
$=$ _____	Substitute.
$=$ _____	Multiply.

2. Make a table of solutions and sketch the graph of the function.

l	10	20	30	40	50
P					

3. As x increases by 10, y increases by _____. So, the function changes at a _____ and therefore is _____. Because the graph _____ from left to right, the function is _____.

8.3 Using Intercepts

Goal: Use *x*- and *y*-intercepts to graph linear equations.

Vocabulary

x-intercept: ☐

y-intercept: ☐

Finding Intercepts

To find the *x*-intercept of a line, substitute ☐ for *y* in the line's equation and solve for ☐.

To find the *y*-intercept of a line, substitute ☐ for *x* in the line's equation and solve for ☐.

Example 1 ***Finding Intercepts of a Graph***

Find the intercepts of the graph of $2x - 5y = -10$.

To find the *x*-intercept, let $y =$ ☐ and solve for *x*.

$2x - 5y = -10$	Write original equation.
$2x - 5(\square) = -10$	Substitute for *y*.
$\square = -10$	Simplify.
$x = \square$	Divide each side by ☐.

To find the y-intercept, let $x =$ ☐ and solve for y.

$2x - 5y = -10$	Write original equation.
$2(\square) - 5y = -10$	Substitute for x.
$\square = -10$	Simplify.
$y = \square$	Divide each side by ☐.

The intercepts of a graph are numbers, not ordered pairs.

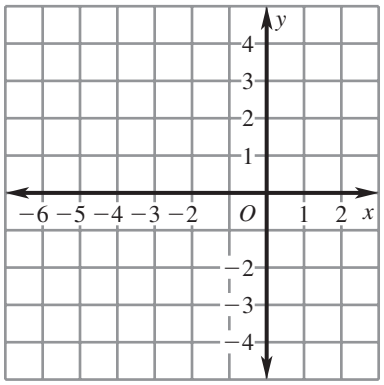
Answer: The x-intercept is ☐, and the y-intercept is ☐.

Example 2 *Using Intercepts to Graph a Linear Equation*

Graph the equation $2x - 5y = -10$ from Example 1.

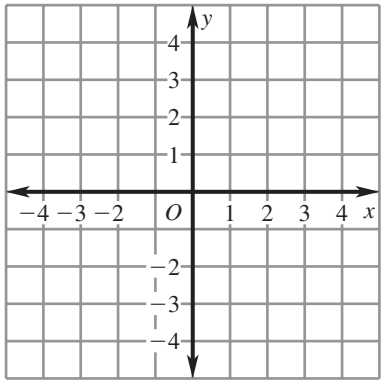
The x-intercept is ☐, so plot the point (☐, 0). The y-intercept is ☐, so plot the point (0, ☐).

Draw a line through the two points.

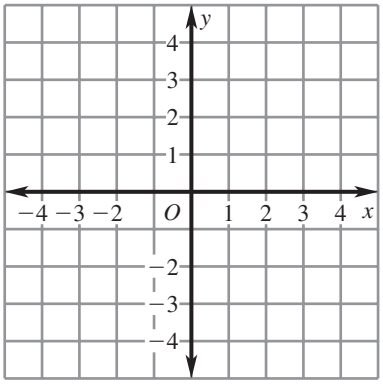


✔ ***Checkpoint*** **Find the intercepts of the equation's graph. Then graph the equation.**

1. $2x + 3y = 6$



2. $3x - 6y = 12$



Example 3 *Writing and Graphing an Equation*

Fitness You run and walk on a fitness trail that is 12 miles long. You can run 6 miles per hour and walk 3 miles per hour. Write and graph an equation describing your possible running and walking times on the fitness trail. Give three possible combinations of running and walking times.

Solution

1. To write an equation, let x be the running time and let y be the walking time (both in hours). First write a verbal model.

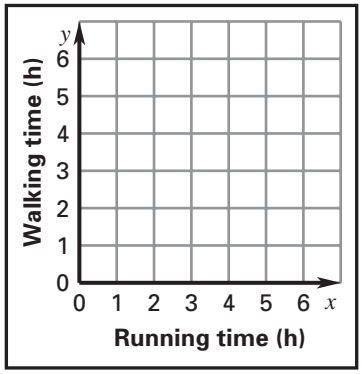
Running time		Walking time		
	•		+		•		=	

Then use the verbal model to write the equation.

2. To graph the equation, find and use the intercepts.

Find x-intercept:

Find y-intercept:



3. Three points on the graph are _____. So, you can _____ _____.

8.4 The Slope of a Line

Goal: Find and interpret slopes of lines.

Vocabulary

Slope:

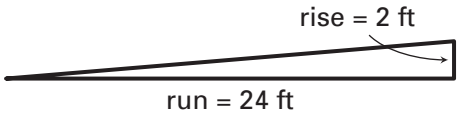
Rise:

Run:

Example 1 ***Finding Slope***

A building's access ramp has a rise of 2 feet and a run of 24 feet. Find its slope.

$$\text{slope} = \frac{\text{rise}}{\square} = \square = \square$$



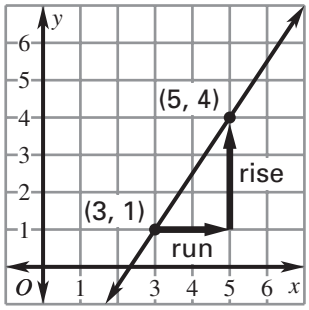
Answer: The access ramp has a slope of $\square$.

Slope of a Line

Given two points on a nonvertical line, you can find the slope m of the line using this formula.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}$$



Example $m = \frac{4 - 1}{5 - 3} = \square$

Example 2 *Finding Positive and Negative Slope*

Find the slope of the line shown.

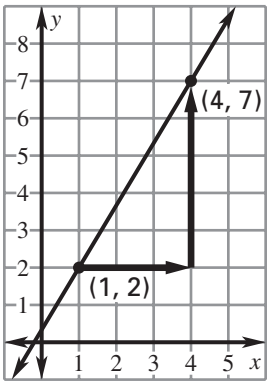
When you calculate a slope, be sure to use the *x*- and *y*-coordinates of the two points in the same order.

a. $m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}$

$= \square$

$= \square$



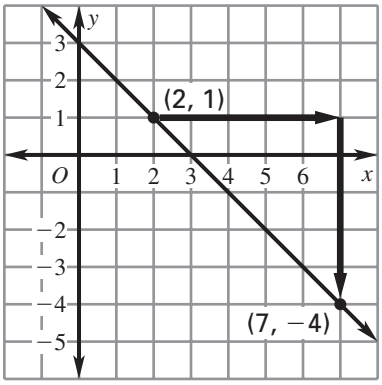
Answer: The slope is $\square$.

b. $m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}$

$= \square$

$= \square = \square$



Answer: The slope is $\square$.

✔ ***Checkpoint*** **Find the slope of the line through the given points.**

1. (2, −2), (0, 4)	**2.** (7, 5), (3, 2)	**3.** (−2, 4), (6, 2)

Example 3 *Zero and Undefined Slope*

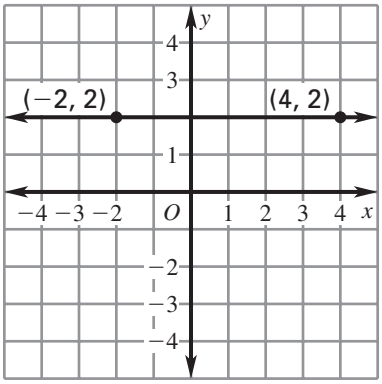
Find the slope of the line shown.

a. $m = \frac{\text{rise}}{\text{run}}$

$= \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}$

$= \boxed{}$

$= \boxed{} = \boxed{}$



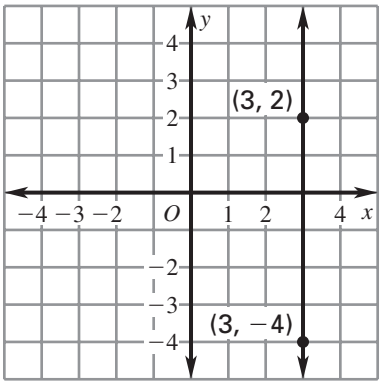
Answer: The slope is $\boxed{}$.

b. $m = \frac{\text{rise}}{\text{run}}$

$= \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}$

$= \boxed{}$

$= \boxed{}$



Answer: The slope is $\boxed{}$.

✔ ***Checkpoint*** **Find the slope of the line through the given points. Tell whether the slope is *positive*, *negative*, *zero*, or *undefined*.**

4. (3, −1), (3, 5)	**5.** (−2, 5), (3, 4)	**6.** (1, −1), (7, −1)

Focus On Graphing

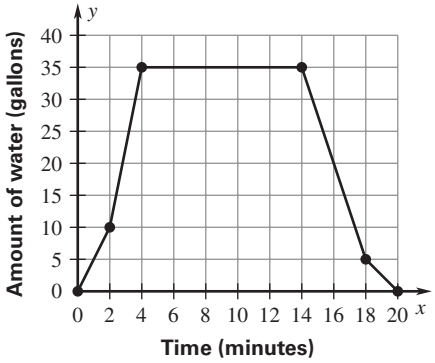
Use after Lesson 8.4

Real-World Graphs

Goal: Interpret and create graphs representing real-world situations.

Example 1 ***Interpreting a Real-World Graph***

Bathtub The graph shows the amount of water in a bathtub. Describe what is happening over time.



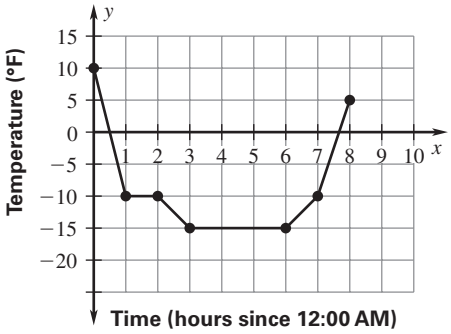
Solution

The slopes of the segments in the graph represent the rates of change in the _____ over ____.

- **First 4 minutes:** The first two segments have _____ slopes, so the bathtub is filling with water. The first segment is not as steep as the second, so the rate at which the water is filling the tub is _____ between 0 and 2 minutes than between 2 and 4 minutes.
- **Next 10 minutes:** The slope of the third segment is 0, so the _____ is not changing. The water has been shut off.
- **After 14 minutes:** The last two segments have _____ slopes, so the water is draining from the bathtub. The fourth segment is steeper than the fifth segment, so the rate at which the water is draining from the tub is _____ between 14 and 18 minutes than between 18 and 20 minutes.

Example 2 *Interpreting a Real-World Graph*

Temperature The graph shows the temperatures on a winter night from midnight until early the next morning. Describe what is happening.



Solution

The first four segments of the graph have alternating _____ and ___ slopes, so the temperature decreases, holds steady, decreases, and then holds steady again. The last two segments of the graph have _____ slopes, so the temperature _____ as morning approaches.

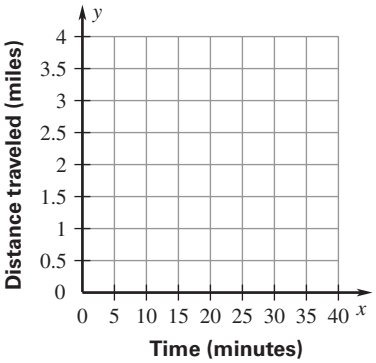
Example 3 *Creating a Real-World Graph*

Weekend Fun One Saturday, a student starts from home and rides a bicycle 3 miles to a friend's house. After visiting for 5 minutes, the friends walk 1 mile to the park. Draw a graph representing this situation.

Solution

A reasonable biking speed would be 12 miles per hour, or 5 minutes per mile. A reasonable walking speed would be 3 miles per hour, or 20 minutes per mile.

The distance from home _____ when the student travels 3 miles (___ minutes of biking) to the friend's house, remains the same for 5 minutes, and then _____ when the friends walk 1 mile (___ minutes of walking) to the park.



8.5 Slope-Intercept Form

Goal: Graph linear equations in slope-intercept form.

Slope-Intercept Form

Words A linear equation of the form $y = mx + b$ is said to be in **slope-intercept form**. The [ ] is m and the [ ] is b.

Algebra $y = mx + b$ **Numbers** $y = 2x + 3$

Example 1 ***Identifying the Slope and y-Intercept***

Identify the slope and y-intercept of the line.

a. $y = 2x - 3$ **b.** $4x + 3y = 9$

Solution

a. Write the equation $y = 2x - 3$ as [ ].

Answer: The line has a slope of [ ] and a y-intercept of [ ].

b. Write the equation $4x + 3y = 9$ in slope-intercept form.

$4x + 3y = 9$	Write original equation.
$3y =$ [ ]	Subtract [ ] from each side.
$y =$ [ ]	Multiply each side by [ ].

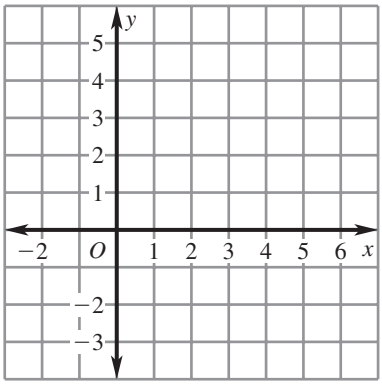
Answer: The line has a slope of [ ] and a y-intercept of [ ].

✔ ***Checkpoint*** **Identify the slope and y-intercept of the line with the given equation.**

1. $y = -3x - 4$	**2.** $x - 2y = 10$

Example 2 *Graphing an Equation in Slope-Intercept Form*

Graph the equation $y = -\frac{3}{4}x + 2$.



1. The *y*-intercept is ☐, so plot the point ☐.

2. The slope is ☐ $= \frac{-\square}{\square}$

Starting at ☐, plot another point by moving right ☐ units and down ☐ units.

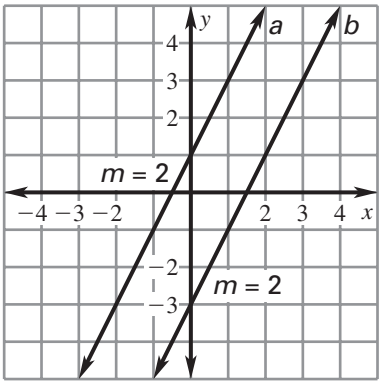
3. Draw a line through the two points.

Slopes of Parallel and Perpendicular Lines

If m is any nonzero number, then the negative reciprocal of m is $-\frac{1}{m}$. Note that the product of a number and its negative reciprocal is -1:

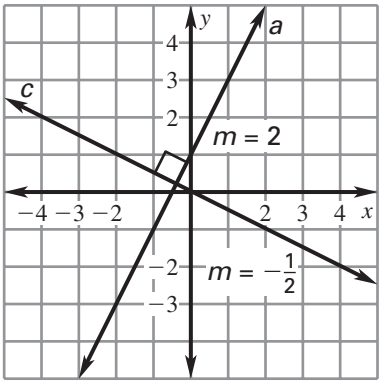
$$m\left(-\frac{1}{m}\right) = -1$$

Two nonvertical parallel lines have ☐. For example, the parallel lines *a* and *b* below ☐.



$a \parallel b$

Two nonvertical perpendicular lines, such as lines *a* and *c* below, have slopes that are ☐.



$a \perp c$

Example 3 *Finding Slopes of Parallel and Perpendicular Lines*

Find the slope of a line that has the given relationship to the line with the equation $5x + 2y = 10$.

a. Parallel to the line **b.** Perpendicular to the line

Solution

a. First write the given equation in slope-intercept form.

$5x + 2y = 10$	Write original equation.
$2y = \square + 10$	Subtract $\square$ from each side.
$y = \square$	Multiply each side by $\square$.

The slope of the given line is $\square$. Because parallel lines have $\square$, the slope of the parallel line is also $\square$.

b. From part (a), the slope of the given line is $\square$. So, the slope of a line perpendicular to the given line is $\square$ $\square$ of $\square$, or $\square$.

✔ ***Checkpoint*** **For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.**

3. $y = -5x - 4$	**4.** $2x - 3y = 6$

Focus On Graphing

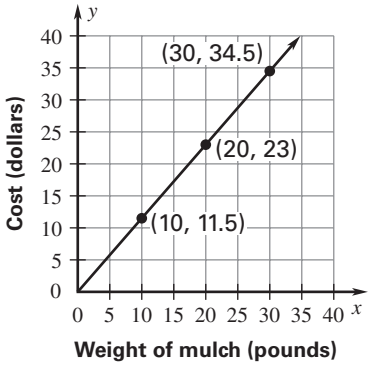
Use after Lesson 8.5

Graphs of Direct Variations

Goal: Analyze a direct variation graph, and graph a direct variation equation.

Example 1 ***Analyzing a Graph***

Buying Mulch The graph shows the cost of buying mulch for landscaping. Tell whether the graph represents a direct variation. If so, tell which variable varies directly with the other. Also identify the constant of variation and interpret it both in relation to the graph and in relation to the real-world situation.



Solution

Find the ratio $\frac{y}{x}$ for each ordered pair ☐ shown on the graph:

$\frac{11.5}{10} =$ ☐ $\frac{23}{20} =$ ☐ $\frac{34.5}{30} =$ ☐

Because the ratios are ☐, the graph represents a ☐. Because *y* represents cost and *x* represents the ☐ of the mulch, the graph shows that ☐ varies directly with the ☐ of the mulch. The constant of variation is ☐. Because (0, 0) is a point on the graph, calculating the slope between this point and any other point on the graph results in the ☐. For instance:

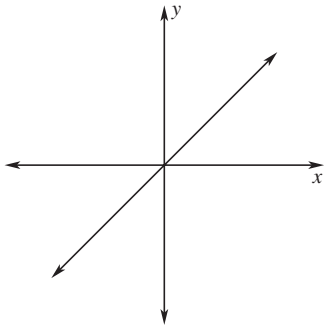
$\frac{11.5-0}{10-0} = \frac{11.5}{10} =$ ☐

In relation to the graph, the constant of variation is the ☐. In relation to the real-world situation, the constant of variation is the ☐ per ☐ of mulch, or ☐ per ☐.

The graph of $y = kx$ is in slope-intercept form, $y = mx + b$. In this case, $m = k$ and $b = 0$, so the graph of $y = kx$ is a line having a slope of k and a y-intercept of 0.

Properties of Graphs of Direct Variations

- A direct variation is defined by the equation $\frac{y}{\square} = k$ for a nonzero constant k. An equivalent form of this equation (when solved for y) is $y = \square$.
- The graph of $y = kx$ is a line through the origin with slope $\square$.



Example 2 ***Drawing and Using a Direct Variation Graph***

Running A runner is training for a race. In the direct variation equation $y = 160x$, y represents the distance traveled (in meters) and x represents the running time (in minutes).

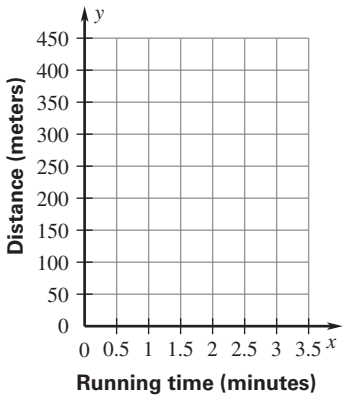
a. Graph the equation. Interpret the graph's slope.

b. The running path is about 400 meters long. Use your graph to estimate the time it will take the runner to run the entire path.

Solution

a. You know that (0, 0) is one point on the graph. Another point on the graph is $\square$. Plot the points and draw a line through them. The slope of the line is $\square$, which represents the runner's average speed, $\square$ meters per minute.

b. Locate $\square$ on the y-axis. Move $\square$ to the graphed line and then $\square$ to the x-axis. You end up at $x = \square$, so the time it takes for the runner to run the entire path is $\square$ minutes.



Focus On Functions

Use after Lesson 8.5

Fitting Lines to Data

Goal: Use the equation of a linear model to solve problems in the context of bivariate measurement data.

Example 1 ***Finding a Linear Model***

Fundraiser The table shows the amounts x (in dollars) spent on advertising and the numbers y of tickets sold for six football games.

Advertising, x	\$750	\$700	\$600	\$500	\$300	\$0
Tickets, y	778	775	754	726	688	600

a. Make a scatter plot of the data.

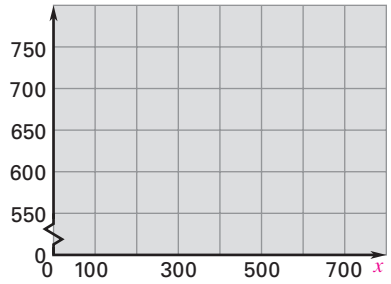
b. Find a linear model for the data.

c. Predict the number of tickets sold for a game when \$200 is spent on advertising.

Solution

a. Write the data as ordered pairs and plot them as points in a coordinate plane.

(750, 778), (700, ☐), (600, ☐), (☐, 726), (300, ☐), (☐, 600)



b. Use the points (0, 600) and (700, 775) to draw a line as close as possible to the data points. Use the points to find the slope of the line.

$m = \frac{\square}{\square} = \frac{\square}{\square}$ or ☐

So, a linear model for the data is $y =$ ☐.

c. Use the linear model from part (b).

$y =$ ☐ Write linear model.

$y =$ ☐ Substitute ☐ for x.

$y =$ ☐ Simplify.

Answer: You can predict that when \$200 is spent on advertising, about ☐ tickets will be sold.

Example 2 *Interpreting Slope and y-Intercept*

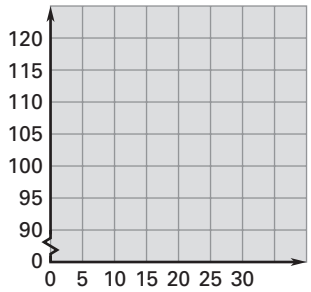
Flying The actual speed y (in knots) of an airplane flying into a headwind of x knots is given by $y = -x + 120$.

a. Graph the equation.

b. Interpret the slope and y-intercept.

Solution

a. The y-intercept is 120, so plot (☐, ☐). The slope is ☐, so move 5 units to the right and ☐ units down, and plot a point at (5, ☐). Then draw a line through the two points.



b. The slope of ☐ means that each mile per hour of headwind decreases the speed of the plane by ☐ mile per hour. The y-intercept of 120 means that the plane flies at ☐ miles per hour when ☐.

✔ ***Checkpoint*** **Complete the following exercise.**

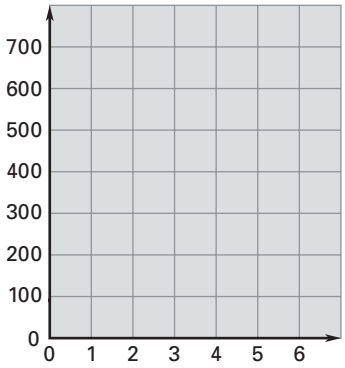
1. The table shows the value y of a product after x years of use.

Age, x	1	2	3	4	5	6
Value, y	700	620	525	480	380	300

a. Make a scatter plot of the data.

b. Find a linear model for the data.

c. Interpret the slope and y-intercept of the graph of the linear model.



8.6 Writing Linear Equations

Goal: Write linear equations.

Vocabulary

Best-fitting line: ☐

Example 1 ***Writing an Equation Given the Slope and y-Intercept***

Write an equation of the line with a slope of −2 and a *y*-intercept of −5.

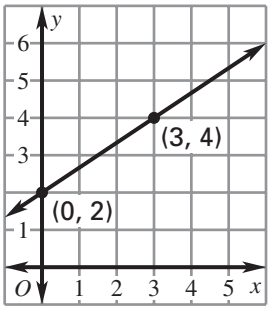
$y = mx + b$	Write general slope-intercept equation.
$y = \square x + \square$	Substitute for m and for b.
$y = \square$	Simplify.

✔ ***Checkpoint***

1. Write an equation of the line with a slope of 4 and a *y*-intercept of −3.

Example 2 ***Writing an Equation of a Graph***

Write an equation of the line shown.



1. Find the slope m using the labeled points.

$m = \square = \square$

2. Find the *y*-intercept b. The line crosses the ☐ at ☐, so $b = \square$.

3. Write an equation of the form $y = mx + b$.

$y = \square x + \square$

Example 3 *Writing Equations of Parallel or Perpendicular Lines*

a. Write an equation of the line that is parallel to the line $y = 8x$ and passes through the point (0, 3).

b. Write an equation of the line that is perpendicular to the line $y = -\frac{1}{2}x + 3$ and passes through the point (0, −5).

Solution

a. The slope of the given line is ☐, so the slope of the parallel line is also ☐. The parallel line passes through (0, 3), so its y-intercept is ☐.

Answer: An equation of the line is ☐.

b. Because the slope of the given line is ☐, the slope of the perpendicular line is the negative reciprocal of ☐, or ☐. The perpendicular line passes through (0, 5), so its y-intercept is ☐.

Answer: An equation of the line is ☐.

✔ ***Checkpoint***

2. Write an equation of the line through the points (0, −3) and (4, 5).
3. Write an equation of the line that is parallel to $y = 3x + 2$ and passes through the point (0, 4).
4. Write an equation of the line that is perpendicular to $y = 3x + 2$ and passes through the point (0, −2).

Example 4 *Approximating a Best-Fitting Line*

Teachers The table shows the number of elementary and secondary school teachers in the United States for the years 1992–1999.

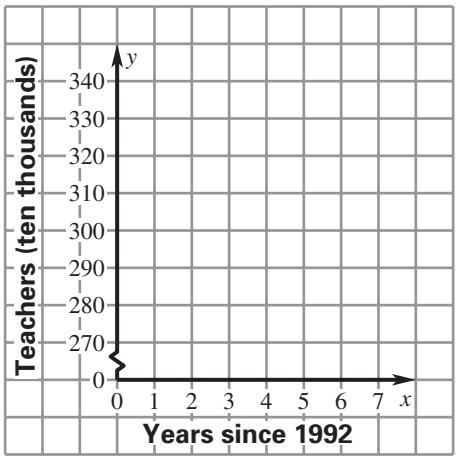
Years since 1992, *x*	0	1	2	3	4	5	6	7
Teachers (in ten thousands), *y*	282	287	293	298	305	313	322	330

a. Approximate the equation of the best-fitting line for the data.

b. Predict the number of teachers in 2006.

Solution

a. *First*, make a scatter plot of the data pairs.



Next, draw the line that appears to best fit the data points. There should be about the same number of points above the line as below it. The line does not have to pass through any of the data points.

Finally, write an equation of the line. To find the slope, estimate the coordinates of two points on the line, such as (0, 280) and (7, 330).

$m = \boxed{} = \boxed{} \approx \boxed{}$

The line intersects the *y*-axis at $\boxed{}$, so the *y*-intercept is $\boxed{}$.

Answer: An approximate equation of the best fitting line is $y = \boxed{}$.

b. Note that 2006 – 1992 = $\boxed{}$, so 2006 is $\boxed{}$ years after 1992.

Calculate *y* when $x = \boxed{}$ using the equation from part (a).

$y = \boxed{} \approx \boxed{}$

Answer: In 2006, there would have been about $\boxed{}$ teachers in the United States.

Focus On Functions

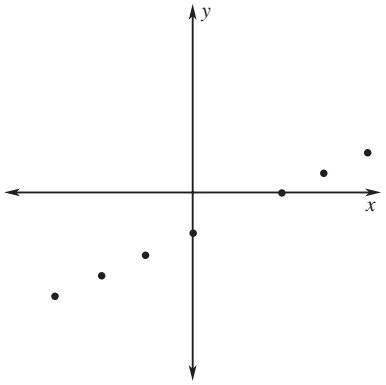
Use after Lesson 8.6

Domain and Range of a Function

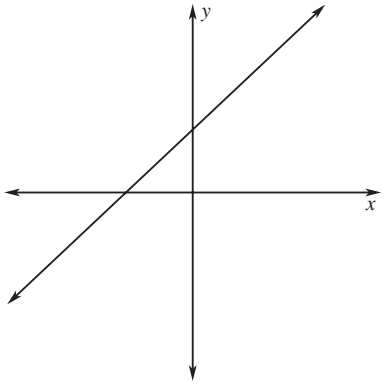
Goal: Analyze the domain and range of a linear function.

Identifying Discrete and Continuous Functions

A _____ has a graph that consist of isolated points.



A _____ has a graph that is unbroken.



As a general rule, you can tell that a function is continuous if you do not have to lift your pencil from the paper to draw its graph.

Example 1 ***Graphing and Classifying a Function***

Graph the function $y = x - 2$ with the given domain. Classify the function as discrete or continuous. Then identify the range.

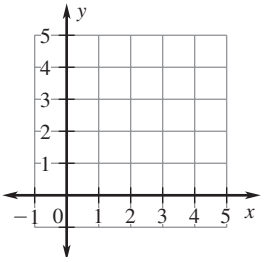
a. Domain: 3, 4, 5

b. Domain: $x \leq 0$

Solution

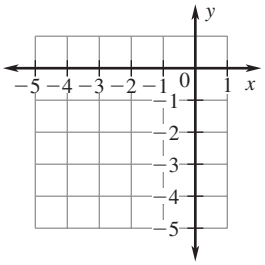
a. Make a table and plot the points.

x			
y			



The function is _____. The range is _____.

b. Plot (0, −2) and draw the part of the line $y = x - 2$ that lines in Quadrant III. The y-values _____ from _____ as x _____ from _____.



The graph is continuous. The range is _____.

When writing an equation for a real-world linear function, you may find it helpful to draw a diagram or use a model.

Example 2 *Classifying a Real-World Function*

Write an equation for the function described. Tell whether the function is *discrete* or *continuous*. Then answer the question.

Exercise An athlete burns 180 calories lifting free weights and then burns 12 calories per minute on the elliptical machine. The total number of calories burned is a function of the number of minutes the athlete spends on the elliptical machine. How many minutes does the athlete need to spend on the elliptical machine if the goal is to burn a total of 420 calories?

Solution

1. Let x represent the number of minutes the athlete spends on the elliptical machine. Let y represent the total number of calories burned. The athlete has already burned ☐ calories and will burn ☐ calories per minute on the elliptical machine, so the equation for the total number of calories burned is given by:

   ☐ = ☐

2. You can burn fractional parts of calories, so the domain consists of ☐. So the function is ☐.

3. To find the number of minutes the athlete needs to spend on the elliptical machine, substitute ☐ for y.

☐ ☐	Substitute.
☐ ☐	Subtract ☐ from each side.
☐	Divide each side by ☐.
Answer	☐ minutes

Focus On Functions

Use after Lesson 8.6

Interpreting Scatter Plots

Goal: Interpret scatter plots.

Review Vocabulary

Positive linear Association:	
Negative linear association:	
No linear Association:	
Outlier:	
Non linear association:	

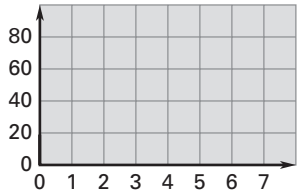
Example 1 ***Analyzing a Scatter Plot***

The table shows the height (in feet) and the price (in dollars) of several ornamental trees. Make a scatter plot of the data. Identify any outliers and describe the association.

Height (feet)	3	3	4	5	5	6	6	7
Price (dollars)	45	50	54	30	63	70	75	80

Solution

Let x represent the height and let y represent the price. Plot the ordered pairs in a coordinate plane.



The data point (_____) is widely separated from the rest of the data. So, it is an _____.

Draw a line through the clustered points. They _____ from left to right, so the data have a _____ association.

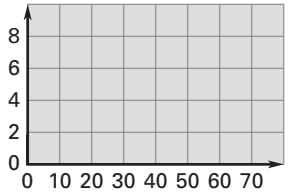
Example 2 *Analyzing a Scatter Plot*

The table shows the age (in years) and the number of visits to the doctor for several people in the past year. Make a scatter plot of the data. Describe the association.

Age	1	2	4	11	21	32	45	62	65	74
Visits	8	6	3	1	1	2	2	4	5	9

Solution

Let *x* represent the age and let *y* represent the number of visits. Plot the ordered pairs in a coordinate plane.



There is no ____ that the points cluster along, but they do cluster around a ____. Draw the ____.

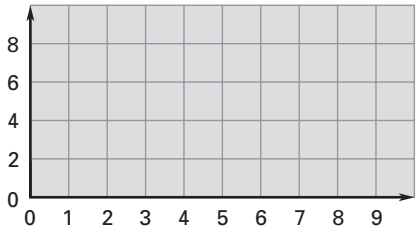
Because the points cluster around a ____, the data have a ____ association.

✔ ***Checkpoint*** **Make a scatter plot of the data in the table. Identify any outliers and describe the association of the data.**

1.

x	1	2	2	3	4	5	6	6	7	9
y	8	8	7	6	5	5	4	3	3	1

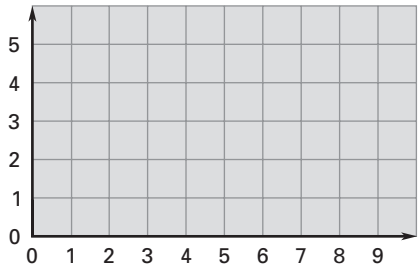
no outliers; negative linear association



2.

x	1	2	3	4	5	5	6	7	8	9
y	1	3	4	5	1	5	5	4	3	2

outlier: (5, 1); nonlinear association



8.7 Function Notation

Goal: Use function notation.

Vocabulary

Function notation: ☐

Example 1 ***Working with Function Notation***

Let $f(x) = 2x - 5$. Find $f(x)$ when $x = -3$, and find x when $f(x) = 13$.

a. $f(x) = 2x - 5$ Write function.

$f(\square) = 2(\square) - 5$ Substitute for x.

$= \square$ Simplify.

Answer: When $x = -3$, $f(x) = \square$.

b. $f(x) = 2x - 5$ Write function.

$\square = 2x - 5$ Substitute for $f(x)$.

$\square = 2x$ Add $\square$ to each side.

$\square = x$ Divide each side by $\square$.

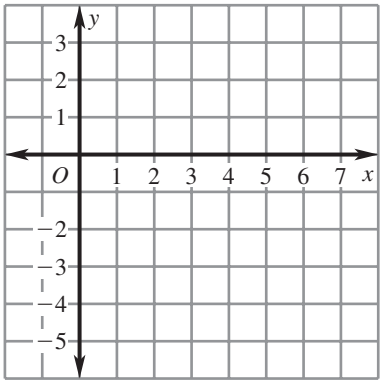
Answer: When $f(x) = 13$, $x = \square$.

✔ ***Checkpoint*** **Let $g(x) = -x + 7$. Find the indicated value.**

1. $g(x)$ when $x = 4$	**2.** x when $g(x) = 9$

Example 2 *Graphing a Function*

Graph the function $f(x) = \frac{5}{6}x - 3$.



1. Rewrite the function as ☐.

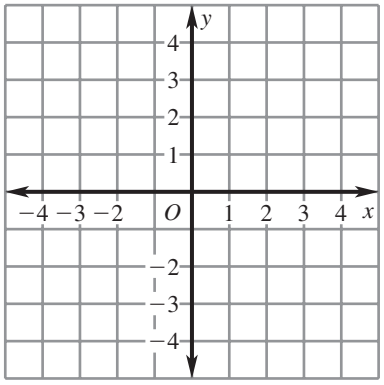
2. The *y*-intercept is ☐, so plot the point ☐.

3. The slope is ☐. Starting at ☐, plot another point by moving right ☐ units and up ☐ units.

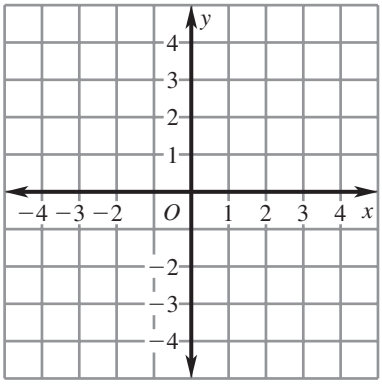
4. Draw a line through the two points.

✔ ***Checkpoint*** **Graph the function.**

3. $g(x) = -\frac{2}{3}x + 2$



4. $h(x) = \frac{3}{2}x - 1$



Example 3 *Writing a Function*

Write a linear function g given that $g(0) = 10$ and $g(4) = -2$.

1. Find the slope m of the function's graph. From the values of $g(0)$ and $g(4)$, you know that the graph of g passes through the points ☐ and ☐. Use these points to calculate the slope.

   $m =$ ☐ $=$ ☐ $=$ ☐

2. Find the y-intercept b of the function's graph. The graph passes through ☐, so $b =$ ☐.

3. Write an equation of the form $g(x) = mx + b$.

   ☐

Example 4 *Using Function Notation in Real Life*

You ride your bike at a speed of 12 miles per hour.

a. Use function notation to write an equation giving the distance traveled as a function of time.

b. How long will it take you to travel 30 miles?

Solution

a. Let t be the elapsed time (in hours) since you started riding your bike, and let $d(t)$ be the distance traveled (in miles) at that time. Write a verbal model. Then use the verbal model to write an equation.

   ☐ $=$ ☐ $\cdot$ ☐

   ☐ $=$ ☐

b. Find the value of t for which $d(t) = 30$.

$d(t) =$ ☐	Write function for distance.
☐ $=$ ☐	Substitute for $d(t)$.
☐ $= t$	Divide each side by ☐.

Answer: It will take you ☐ hours to travel 30 miles.

Focus On Functions

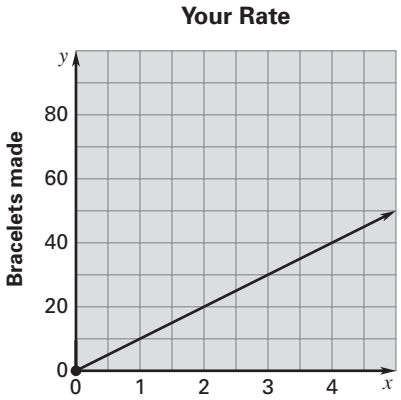
Use after Lesson 8.7

Properties of Functions

Goal: Compare two different proportional relationships and compare two different functional relationships.

Example 1 *Comparing Proportional Relationships*

The graph and table show the number of bracelets *y* you and your friend can make in *x* hours, respectively. Who can make bracelets faster? Explain.



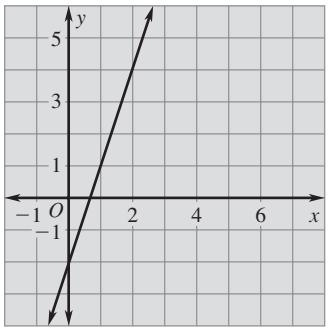
Your Friend's Rate	
Hours, *x*	**Bracelets, *y***
2	10
4	20
6	30
8	40

Solution

The slope of the line shown in the graph is ____. So, you can make bracelets at a rate of ____. The data in the table vary directly. The constant of variation is ____. So, your friend can make bracelets at a rate of ____. ____ can make bracelets faster.

Example 2 *Comparing Functions*

Compare the function $f(x) = \frac{1}{3}x + 2$ with the linear function shown in the graph.



Solution

The slope of $f(x) = \frac{1}{3}x + 2$ is ____ and the *y*-intercept is ____. The slope of function shown in the graph is ____ and the *y*-intercept is ____. The function shown in the graph has a ____ rate of change because its slope is ____.

Example 3 *Comparing Real-World Functions*

You and your friend each want to buy a shool jacket that costs $75. You can save $13 per week, starting now. The table shows what your friend can save. (a) Compare your savings plan to that of your friend. (b) Who can buy the jacket first? Explain.

Weeks	Amount saved
0	$25
1	$36
2	$47
3	$58
4	$69
5	$80

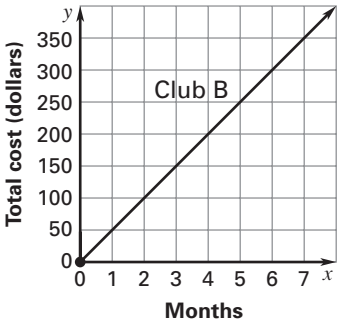
Solution

a. The table shows that your friend starts with $☐ and saves $☐ per week. You start with $☐ and save $☐ per week. So, ☐ save at a greater rate.

b. It will take you ☐ weeks ($☐ × ☐ = $☐) to save enough money. Using the table, it will take your friend ☐ weeks to save enough money. So, ☐ will be able to buy the jacket first.

✔ ***Checkpoint*** **Complete the following exercise.**

1. Membership Your family wants to join a fitness club. The cost to join club A for x months is given by the function $y = 45x$. The cost to join club B for x months is shown in the graph. Club C charges a $75 initiation fee. The cost to join club C for x months is shown in the table.



Club C	
Months, *x*	**Cost, *y***
1	$100
2	$125
3	$150
4	$175

a. Compare the monthly rates of club A and club B.
b. Compare the monthly rates of club B and club C.
c. Which club costs the least to join for 10 months?

8.8 Systems of Linear Equations

Goal: Graph and solve systems of linear equations.

Vocabulary

System of linear equations: ☐

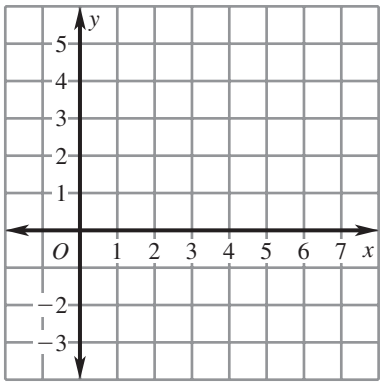
Solution of a linear system: ☐

Example 1 ***Solving a System of Linear Equations***

Solve the linear system: $y = x - 3$ **Equation 1**

$y = -\frac{1}{5}x + 3$ **Equation 2**

1. Graph the equations.



2. Identify the apparent intersection point, ☐.

3. Verify that ☐ is the solution of the system by substituting ☐ for x and ☐ for y in each equation.

Equation 1

$y = x - 3$

☐ $\stackrel{?}{=}$ ☐ $- 3$

☐ ☐ ☐

Equation 2

$y = -\frac{1}{5}x + 3$

☐ $\stackrel{?}{=} -\frac{1}{5}x$ ☐ $+ 3$

☐ ☐ ☐

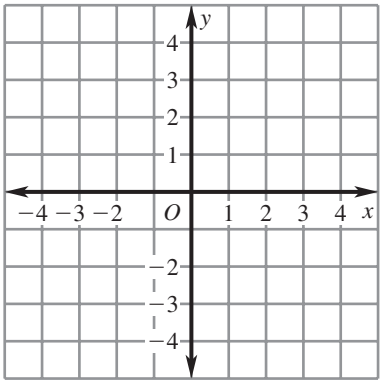
Answer: The solution is ☐.

Example 2 *Solving a Linear System with No Solution*

Solve the linear system: $y = -3x - 2$ **Equation 1**

$y = -3x + 3$ **Equation 2**

Graph the equations. The graphs appear to be [ ] lines. You can confirm that the lines are [ ] by observing from their equations that they have the [ ] slope, [ ], but [ ] y–intercepts, [ ] and [ ].



Answer: Because parallel lines [ ], the linear system has [ ].

Example 3 *Solving a Linear System with Many Solutions*

Solve the linear system: $2x + y = 4$ **Equation 1**

$-6x + 3y = -12$ **Equation 2**

Write each equation in slope-intercept form and then graph the equations.

Equation 1

$2x + y = 4$

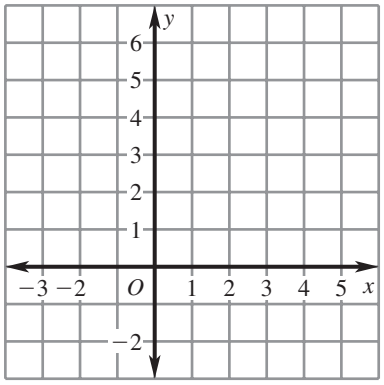
$y =$ [ ]

Equation 2

$-6x - 3y = -12$

[ ]

$y =$ [ ]

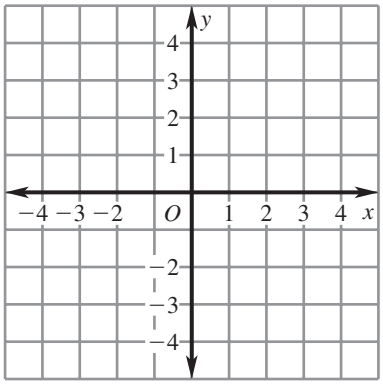


The slope-intercept forms of equations 1 and 2 are identical, so the graphs of the equations are [ ].

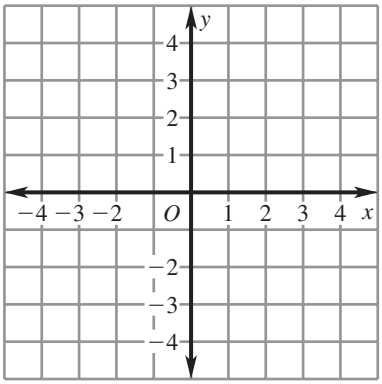
Answer: Because the graphs have infinitely many [ ] [ ], the system has [ ]. Any [ ] on the line [ ] represents a solution.

Checkpoint Solve the linear system by graphing.

1. $y = -x + 3$
$y = \frac{1}{3}x - 1$



2. $5x + y = -3$
$10x + 2y = 8$



Example 4 ***Writing and Solving a Linear System***

An ecologist is studying the population of two types of fish in a lake. Use the information in the table to predict when the population of the two types of fish will be the same.

Fish type	Current population	Change (number per year)
A	340	−25
B	180	15

Solution

Let y be the number of fish after x years. Write a linear system.

Fish A population: _____

Fish B population: _____

Use a graphing calculator to graph the equations. Trace along one of the graphs until the cursor is on the point of intersection. This point is _____.

Answer: The number of fish will be the same after ____ years when the population of each fish will be ____.

Focus on Functions

Use after Lesson 8.8

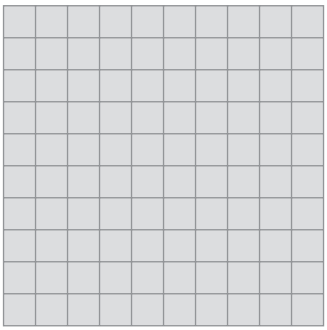
Solving Systems of Equations

Goal: Solve systems of equations graphically and algebraically.

Example 1 *Estimating the Solution of a Linear System*

Graphically estimate the solution of the linear system:

$5x + y = 2$ Equation 1
$-10x - 4y = 3$ Equation 2



Solution

Graph the equations. Because the lines do not intersect at an ☐ of ☐, estimate the point of intersection. The lines appear to intersect at about (☐, ☐).

Answer: The solution of the system is approximately (☐, ☐).

Example 2 *Solving a Linear System Algebraically*

Solve the linear system algebraically:

$5x + y = 2$ Equation 1
$-10x - 4y = 3$ Equation 2

1. Solve one of the equations for one variable.

$5x + y = 2$ Equation 1
$y =$ ☐ Solve for y.

2. Substitute for y in the other equation and solve for x.

$-10x - 4y = 3$ Equation 2
$-10x - 4($☐$) = 3$ Substitute (☐) for y.
$-10x -$ ☐ $+$ ☐ $= 3$ Distributive property
☐$x =$ ☐ Simplify.
$x =$ ☐ Divide each side by ☐.

3. Substitute the value of x in one of the original equations to find y.

$5($☐$) + y = 2$ Substitute ☐ for x in Equation 1.
☐ $+ y = 2$ Multiply.
$y =$ ☐ Subtract ☐ from each side.

Answer: The exact solution of the system is (☐, ☐).

Example 3 *Solving a Linear System by Inspection*

Solve the linear system by inspection.

a. $x + y = 2$
$3x + 3y = 6$

b. $x - y = -1$
$x - y = 4$

c. $x = -5$
$y = 1$

Solution

a. The [ ] property of equality can be used to obtain [ ] from [ ], so the two equations are [ ].

Answer The system has an [ ] number of solutions consisting of all the points on the line [ ].

b. The left side of each equation is the [ ]. It is not possible for the value of $x - y$ to equal both [ ] and [ ].

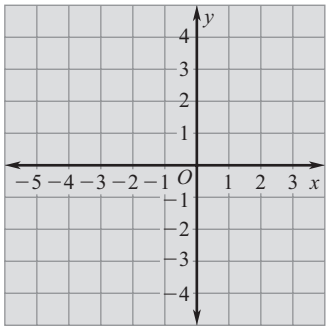
Answer The system has [ ].

c. The vertical line $x = -5$ and the horizontal line $y = 1$ intersect at the point ([ ], [ ]).

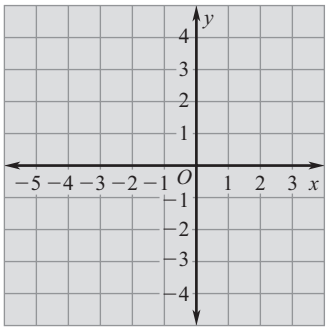
Answer The solution of the system is ([ ], [ ]).

✔ ***Checkpoint*** **Graphically estimate the solution of the linear system. Then solve the system algebraically.**

1. $5x + 20y = -5$
$x - y = -4$



2. $2x - 5y = 3$
$-2x + 10y = -1$



8.9 Graphs of Linear Inequalities

Goal: Graph inequalities in two variables.

Vocabulary

Linear inequality: ☐

Solution of a linear inequality: ☐

Graph of a linear inequality: ☐

Example 1 ***Checking Solutions of a Linear Inequality***

Tell whether the ordered pair is a solution of $3x - y > 2$.

a. (3, 0) **b.** (−1, 5)

Solution

a. Substitute for x and for y.

$$3x - y > 2$$

$$3\square - \square \overset{?}{>} 2$$

$$\square\ \square\ \square$$

(3, 0) ☐ a solution.

b. Substitute for x and for y.

$$3x - y > 2$$

$$3\square - \square \overset{?}{>} 2$$

$$\square\ \square\ \square$$

(−1, 5) ☐ a solution.

✔ ***Checkpoint*** **Tell whether the ordered pair is a solution of $-x + 2y > 4$.**

1. (1, 6)	**2.** (−7, −2)	**3.** (2, 3)

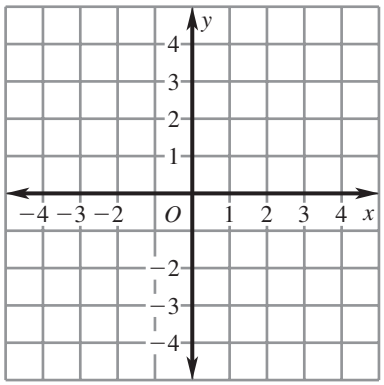
Graphing Linear Inequalities

1. Find the equation of the boundary line by replacing the inequality symbol with =. Graph this equation. Use a dashed line for < or >. Use a solid line for ≤ or ≥.
2. Test a point in one of the half-planes to determine whether it is a solution of the inequality.
3. If the test point is a solution, shade the half-plane that contains the point. If not, shade the other half-plane.

Example 2 ***Graphing a Linear Inequality***

Graph $y \geq -x + 1$.

1. Draw the boundary line $y = -x + 1$. The inequality symbol is ≥, so use a ☐.



2. Test the point (0, 0) in the inequality.

$$y \geq -x + 1$$

$$\square \overset{?}{\geq} \square + 1$$

$$\square \quad \square \quad \square$$

3. Because (0, 0) ☐ a solution, shade the half-plane that ☐.

Example 3 *Graphing Inequalities with One Variable*

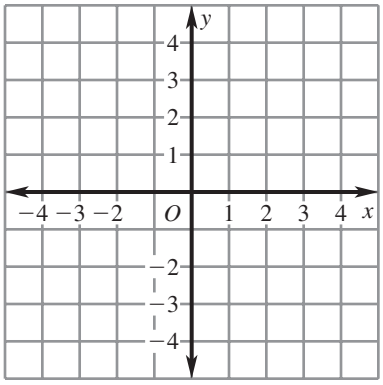
Graph $x > -2$ and $y \leq 3$ in a coordinate plane.

a. Graph $x = -2$ using a [ ] line. Use (0, 0) as a test point.

$x > -2$

[ ] [ ] [ ]

Shade the half-plane [ ].

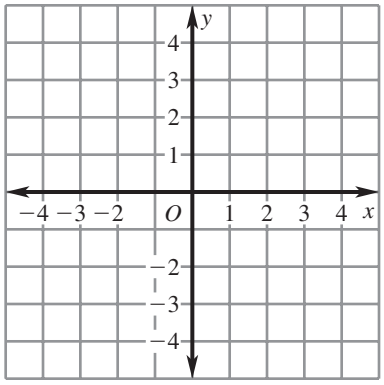


b. Graph $y = 3$ using a [ ] line. Use (0, 0) as a test point.

$y \leq 3$

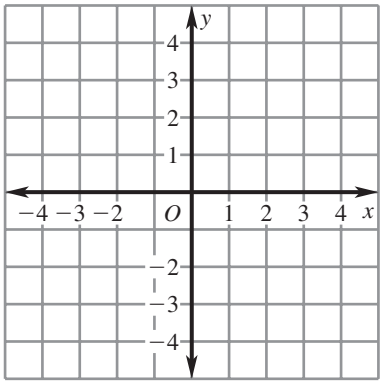
[ ] [ ] [ ]

Shade the half-plane [ ].

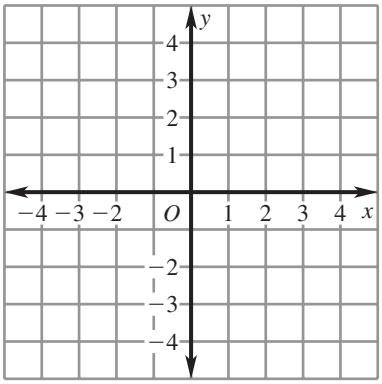


✔ ***Checkpoint*** **Graph the inequality in a coordinate plane.**

4. $6x - 3y < 9$



5. $x \leq 4$



8 Words to Review

Give an example of the vocabulary word.

Relation

Domain

Range

Input

Output

Function

Vertical line test

Equation in two variables

Solution of an equation in two variables

Graph of an equation in two variables

Linear equation

Linear function

Function form

***x*-intercept**

***y*-intercept**

Slope

Rise

Run

Slope-intercept form

Best-fitting line

Discrete function

Continuous function

Function notation

System of linear equations

Solution of a linear system

Linear inequality in two variables

Solution of a linear inequality in two variables

Graph of a linear inequality in two variables

Review your notes and Chapter 8 by using the Chapter Review on pages 466–469 of your textbook.