4.5 Rules of Exponents

Goal: Multiply and divide powers.

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Algebra $a^m \cdot a^n = a^{m+n}$

Numbers $4^3 \cdot 4^2 = 4^{\square} = 4^{\square}$

Example 1 *Using the Product of Powers Property*

a. $4^7 \cdot 4^{11} = 4^{\square}$ Product of powers property

$= 4^{\square}$ Add exponents.

b. $2x^2 \cdot 7x^6 = 2 \cdot 7 \cdot x^2 \cdot x^6$ Commutative property of multiplication

$= 2 \cdot 7 \cdot x^{\square}$ Product of powers property

$= 2 \cdot 7 \cdot x^{\square}$ Add exponents.

$= \square$ Multiply.

✔ ***Checkpoint*** **Find the product. Write your answer using exponents.**

1. $2^5 \cdot 2^{12}$	**2.** $(0.4)^6 \cdot (0.4)^2 \cdot (0.4)^3$
3. $x^6 \cdot x^{13}$	**4.** $b^2 \cdot b^4 \cdot b$

Quotient of Powers Property

Words To divide powers with the same base, subtract the exponent of the denominator from the exponent of the numerator.

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Numbers $\frac{5^7}{5^4} = 5^{\square} = 5^{\square}$

Example 2 *Using the Quotient of Powers Property*

a. $\frac{(0.6)^8}{(0.6)^3} = (0.6)^{\square}$ Quotient of powers property

$= (0.6)^{\square}$ Subtract exponents.

b. $\frac{3x^7}{12x^3} = \frac{3x^{\square}}{12}$ Quotient of powers property

$= \frac{3x^{\square}}{12}$ Subtract exponents.

$= \square$ Divide numerator and denominator by $\square$.

✔ ***Checkpoint*** **Find the quotient. Write your answer using exponents.**

5. $\frac{5^9}{5^2}$	**6.** $\frac{(1.4)^7}{(1.4)^4}$
7. $\frac{4x^{13}}{24x^9}$	**8.** $\frac{14x^{16}}{6x^{11}}$

Example 3 ***Using Both Properties of Powers***

Simplify $\frac{4m^3 \cdot m^4}{12m^2}$.

$\frac{4m^3 \cdot m^4}{12m^2} = \frac{4m^{\square}}{12m^2}$ Product of powers property

$= \frac{4m^{\square}}{12m^2}$ Add exponents.

$= \frac{4m^{\square}}{12}$ Quotient of powers property

$= \frac{4m^{\square}}{12}$ Subtract exponents.

$= \square$ Divide numerator and denominator by $\square$.

Checkpoint **Simplify.**

9. $\frac{6m^5 \cdot m}{15m^3}$	**10.** $\frac{n^2 \cdot 10n^6}{5n^3}$

4.6 Negative and Zero Exponents

Goal: Work with negative and zero exponents.

Negative and Zero Exponents

For any nonzero number a, $a^0 = 1$.

For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

Example 1 ***Powers with Negative and Zero Exponents***

Write the expression using only positive exponents.

a. $4^{-3} =$ ☐ Definition of negative exponent

b. $m^{-5}n^0 = m^{-5} \cdot$ ☐ Definition of zero exponent

$=$ ☐ Definition of negative exponent

c. $13xy^{-8} =$ ☐ Definition of negative exponent

✔ ***Checkpoint*** **Write the expression using only positive exponents.**

1. $33{,}333^0$	**2.** 7^{-3}	**3.** $2z^{-2}$	**4.** $3x^{-4}y$

Example 2 ***Rewriting Fractions***

Write the expression without using a fraction bar.

a. $\frac{1}{15} =$ ☐ Definition of negative exponent

b. $\frac{a^3}{c^5} =$ ☐ Definition of negative exponent

Checkpoint **Write the expression without using a fraction bar.**

5. $\frac{1}{18}$	**6.** $\frac{1}{100}$	**7.** $\frac{3}{c^2}$	**8.** $\frac{x^5}{y^7}$

Example 3 ***Using Powers Properties with Negative Exponents***

Find the product or quotient. Write your answer using only positive exponents.

a. $6^{12} \cdot 6^{-4}$ **b.** $\frac{0.7n^{-4}}{n}$

Solution

a. $6^{12} \cdot 6^{-4} = 6^{\square}$ Product of powers property

$= 6^{\square}$ Add exponents.

b. $\frac{0.7n^{-4}}{n} = 0.7n^{\square}$ Quotient of powers property

$= 0.7n^{\square}$ Subtract exponents.

$= \square$ Definition of negative exponent

Checkpoint **Find the product or quotient. Write your answer using only positive exponents.**

9. $(0.3)^{10} \cdot (0.3)^{-7}$	**10.** $\frac{7d^{-4}}{d^2}$

Focus On Numbers

Use before Lesson 4.7

Writing Numbers Using Powers of 10

Goal: Write numbers using powers of 10.

Review Vocabulary

Power: _____

Power of 10	Decimal Equivalent
10^4	☐
10^3	☐
10^2	☐
10^1	☐
☐	1
10^{-1}	☐
10^{-2}	☐
10^{-3}	☐
10^{-4}	☐

Example 1 ***Rewriting a Large Number as a Power of 10***

There are 4,000,000,000 bytes of memory in 4 gigabytes of RAM. Write this number using a power of 10.

4,000,000,000 = ☐ × ☐ Rewrite 4,000,000,000 as ☐ × ☐.

= ☐ × $10^{\square}$ Rewrite ☐ as $10^{\square}$.

Checkpoint **Write the number using a power of 10.**

1. A stadium has 90,000 seats.

2. A grain of sand weighs 0.000002 gram.

Example 2 ***Solving Problems Using Powers of 10***

A bookstore has 40,000 books on its shelves. An online bookseller has 30,000,000 books available. About how many times greater is the number of books available online than the number of books in the store?

Solution

Write each number using the power of 10 of the ☐ number. Then divide the decimal part of the ☐ number by the decimal part of the ☐ number.

On shelves: 40,000 = ☐ × ☐ = ☐ × $10^{\square}$

Online: 30,000,000 = ☐ × ☐ = ☐ × $10^{\square}$

= ☐ × $10^{\square}$

☐ ÷ ☐ = ☐ Divide.

Answer: The number of books available online is about ☐ times greater than the number of books in the store.

Checkpoint **Complete the following exercise.**

3. A housefly weighs about 0.00002 pound. A dime weighs about 0.005 pound. How many times greater is the weight of the dime than the weight of the housefly?

4.7 Scientific Notation

Goal: Write numbers using scientific notation.

Using Scientific Notation

A number is written in **scientific notation** if it has the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer.

Standard form	Product form	Scientific notation
725,000	$7.25 \times 100{,}000$	7.25×10^5
0.006	6×0.001	6×10^{-3}

Example 1 *Writing Numbers in Scientific Notation*

a. The average distance Mars is from the sun is 141,600,000 miles. Write this number in scientific notation.

Standard form	Product form	Scientific notation

b. The diameter of a quarter-ounce gold American Eagle coin is 0.022 meter. Write this number in scientific notation.

Standard form	Product form	Scientific notation

Example 2 *Writing Numbers in Standard Form*

a. Write 4.1×10^4 in standard form.

Scientific notation	Product form	Standard form

b. Write 7.23×10^{-6} in standard form.

Scientific notation	Product form	Standard form

Checkpoint **Write the number in scientific notation.**

1. 3,050,000,000	**2.** 0.000082

Write the number in standard form.

3. 6.53×10^{7}	**4.** 9.2×10^{-4}

Example 3 ***Ordering Numbers Using Scientific Notation***

Order 5.3×10^{5}, 520,000, and 7.5×10^{4} from least to greatest.

1. Write each number in scientific notation if necessary.

520,000 = ☐

2. Order the numbers with different powers of 10.

Because $10^{\square} < 10^{\square}$, ☐ < ☐ and ☐ < ☐.

3. Order the numbers with the same power of 10.

Because ☐ < ☐, ☐ < ☐.

4. Write the original numbers in order from least to greatest.

☐; ☐; ☐

Checkpoint **Order the numbers from least to greatest.**

5. 23,000; 3.4×10^{3}; 2.2×10^{4}
6. 4.5×10^{-4}; 0.000047; 4.8×10^{-5}

Example 4 ***Multiplying Numbers in Scientific Notation***

Oxygen Atoms The volume of one mole of oxygen atoms is about 1.736×10^{-5} cubic meters. Find the volume of 1.5×10^{4} moles of oxygen atoms.

Solution

Total volume $=$ Volume of one mole of oxygen atoms $\times$ Number of moles	
$=$ ( )( )	Substitute values.
$=$ ( )( )	Commutative and associative properties of multiplication
$=$ $\times$ ( )	Multiply and .
$=$ $\times$ ( )	Product of powers property
$=$ $\times$	Add exponents.

Answer: The volume of 1.5×10^{4} moles of oxygen atoms is about $\times$ cubic meters.

✔ ***Checkpoint*** **Find the product. Write your answer in scientific notation.**

7. $(2.5 \times 10^{3})(2 \times 10^{5})$	**8.** $(1.5 \times 10^{-2})(4 \times 10^{-4})$

Focus On Operations

Use after Lesson 4.7

Operations with Scientific Notation

Goal: Perform operations with numbers written in scientific notation.

Review Vocabulary

Scientific notation: ____

Example 1 *Multiplying Numbers Written in Scientific Notation*

Evaluate $(5.6 \times 10^6) \times (4.5 \times 10^2)$. Write your answer in scientific notation.

Solution

$(5.6 \times 10^6) \times (4.5 \times 10^2)$

$= (5.6 \times \square) \times (\square \times 10^2)$ Commutative and associative properties of multiplication

$= \square \times 10^{\square}$ ____ of powers property

$= (\square \times 10^{\square}) \times 10^{\square}$ Rewrite ____.

$= \square \times (10^{\square} \times 10^{\square})$ ____ property of multiplication

$= \square \times 10^{\square}$ ____ of powers property

Checkpoint **Evaluate. Write your answer in scientific notation.**

1. $(5.1 \times 10^{8}) \times (9 \times 10^{-3})$	**2.** $(8.8 \times 10^{5})(2.3 \times 10^{4})$
3. $\dfrac{1.28 \times 10^{-5}}{3.2 \times 10^{2}}$	**4.** $\dfrac{5.68 \times 10^{8}}{8 \times 10^{-1}}$

Example 2 ***Adding Numbers Written in Scientific Notation***

Evaluate $(1.62 \times 10^{2}) + (7.78 \times 10^{3})$. Write your answer in scientific notation.

Solution

Rewrite 7.78×10^{3} so it has the same power of ☐ as ☐.

$7.78 \times 10^{3} = 7.78 \times 10^{\square} \times 10^{\square} = \square \times 10^{\square}$

$(1.62 \times 10^{2}) + (\square \times 10^{\square})$

$= (\square + \square) \times \square$ Distributive property

$= \square \times \square$ Add.

$= (\square \times 10^{\square}) \times 10^{\square}$ Rewrite ☐.

$= \square \times 10^{\square}$ ☐ of powers property

Checkpoint **Evaluate. Write your answer in scientific notation.**

5. $(8.63 \times 10^{9}) + (1 \times 10^{7})$	**6.** $(7 \times 10^{-2}) - (2 \times 10^{-3})$

4 Words to Review

Give an example of the vocabulary word.

Prime number

Composite number

Prime factorization

Factor tree

Monomial

Common factor

Greatest common factor (GCF)

Relatively prime

Equivalent fractions

Simplest form

Multiple

Common multiple

Least common multiple (LCM)

Least common denominator (LCD)

Scientific notation

Review your notes and Chapter 4 by using the Chapter Review on pages 212–215 of your textbook.